Moment of Inertia and Mass Contrasted

Teacher's Guide

Task #2—Determine the Period for the Disk and Ring Pendulums

The author found the following values for the periods:

T_{disk} = 0.642 s

T_{ring} = 0.716 s

With period accuracy to three, the periods are clearly different. The periods different by less than 0.1 s. Therefore, the use of a stop watch would not be advised. The associated start and stop reflex errors could well mask any differences in the periods. Students can easily visualize the difference in periods by placing the disk and ring on the axis and then start swinging them simultaneously. Students will notice that after just a few periods, the disk and ring are out of sync.

Task #3—Compute the Experimental Moments of Inertia

Students must be careful to express all of the parameters in the theoretical equation for the moment of inertia in MKS units.

Using the theoretical equation for moment of inertia, the author found the following experimental values for the moments of inertia about the axes near the edge of the disk and ring:

(Experiment) I_{disk} = 2.02 x 10^{-4} kg-m^2

(Experiment) I_{ring} = 2.45 x 10^{-4} kg-m^2

Task #4—Determine the Theoretical Moments of Inertia

For the ***disk***, the Parallel-axis Theorem, $I = I_c + mw^2$, tells us that:

$$I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2. \qquad \text{Equation 4}$$

For the ***ring***, the Parallel-axis Theorem, $I = I_c + mw^2$, tells us that:

$$I = \frac{M}{2}(R_1^2 + R_2^2) + MR_2^2 = \frac{M}{2}(R_1^2 + 3R_2^2). \qquad \text{Equation 5}$$

Using Equations 4 and 5, the author found the following theoretical values for the moments of inertia about the axes near the edge of the disk and ring:

(Theory) I_{disk} = 1.97 x 10^{-4} kg-m^2

(Theory) I_{ring} = 2.41 x 10^{-4} kg-m^2

Task #5—Determine the Percent Error Between the Experimental and Theoretical Moments of Inertia

The table below summarizes the results as obtained by the author.

Object	**Experimental Moment of Inertia** *(kg-m²)* $I = \frac{T^2 M g d}{4\pi^2}$	**Theoretical Moment of Inertia** *(kg-m²)*	**% Difference** (Expt – Theory)/Theory x 100%
Disk	2.02×10^{-4} kg-m²	1.97×10^{-4} kg-m²	2.5 %
Ring	2.45×10^{-4} kg-m²	2.41×10^{-4} kg-m²	1.7 %

In both cases, the magnitude of the percent difference is less than 2%, showing close agreement between experiment and theory.

Task #6—Compare the Disk and Ring Moments of Inertia and Discuss Reasons for Any Difference

Based upon our error analysis in Task #5, we have good reason to conclude that the moment of inertia of the ring about its edge axis is greater than the moment of inertia of the disk about its edge axis. We would expect this since the period of the ring was noticeably longer than the period of the disk.

If a mass is further from the axis of rotation, it is more difficult to change its angular velocity. Intuition suggests that this is due to the fact the mass is carrying more momentum on the circle and the momentum vector is changing more rapidly. Both of these things depend on how far the mass is from the axis of rotation. For the ring, the majority of mass is concentrated further from the axis than with the disk.