Moment of Inertia Challenge

Teacher's Guide

Introduction to the Moment of Inertia Challenge

There is a link in this lesson to a pdf file that will allow the teacher to print the desired number of copies of the pattern for the card stock disks. The disks can then be cut out from the card stock paper using a pair scissors. Card stock should be used as it is heavier paper that will better support the 16 wood blocks that will be taped to it using double-stick tape. The students should tape two of these disks together back-to-back with their pivot point and magnet locations aligned. This will ensure that the wood blocks are aligned that will be taped to the front and back of the completed disk. The author found the wood blocks shown in Figure 1 at a Michaels hobby shop—at a "regular price" of approximately $5. Alternately, you could use identical dice instead of wood blocks.

Figure 1

***Challenge #1*—Use PocketLab to collect and then analyze data allowing you to determine the period *T* for each of the two physical pendulums. From the period and other measurable parameters, calculate what we will refer to as the "experimental value" for the moment of inertia in kg-m*m of each physical pendulum *about the axis near the edge of the disk*.**

Figure 2 shows a typical graph of magnetic field magnitude vs. time. In order to obtain the best possible accuracy, students should have the data rate set to the maximum possible with PocketLab—50 points/second. They should also compute the time for several periods, and not just look at a single period. In Figure 2, the time for 17 periods was noted as 19.00 seconds, giving a period T = 1.118 s. Using PocketLab provides much greater accuracy than possible with a stop watch, as PocketLab avoids reaction time in starting and stopping the stop watch.

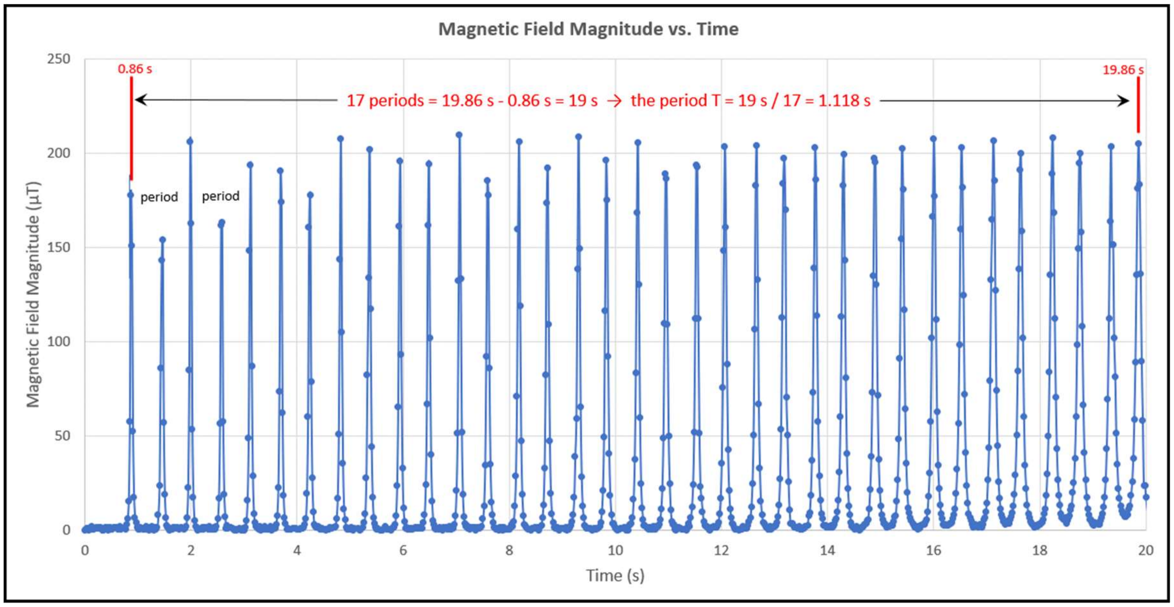
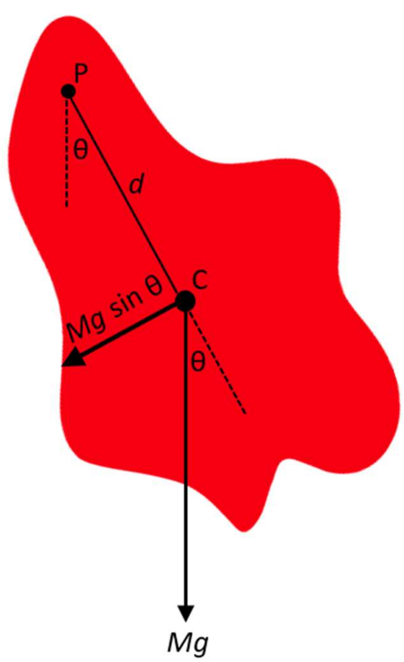


Figure 2

In order to calculate the moment of inertia of each physical pendulum about the pivot point, students will need to make use of an equation that is commonly studied in physics classes. This equation and its derivation are shown in Figure 3.

The Physical Pendulum



The red object is the physical pendulum.
Its mass is M.
P is the pivot point.
C is the center of mass.
Equilibrium is where C is directly below P.
d is the distance from the pivot point to the center of mass.
I is the moment of inertia about an axis through pivot P.
The restoring torque is $\tau = Mgd \sin\theta$.
Since $\theta \approx \sin\theta$ for small θ, then $\tau = Mgd\theta$.
Since $\tau = I\alpha$, where α is the angular acceleration, $\alpha = Mgd\theta/I$.
This is the same form as for a simple pendulum. Therefore,

$$T = 2\pi\sqrt{\frac{I}{Mgd}}$$

Solving for I: $$\boxed{I = \frac{T^2 Mgd}{4\pi^2}}$$

Since T, M, g, and d can be directly measured, the moment of inertia I of the object about the pivot point can be readily determined.

Figure 3

The equation in the box at the bottom of Figure 3 is the result of interest in this challenge. With T, M, and d all measurable and g known, the moment of inertia about the pivot P can easily be calculated. Students should be careful to record all measurements in MKS (Meter, Kilogram, Second) units of measure. This will ensure that the moment of inertia is expressed in kg-m^2. The authors results are as follows:

Physical pendulum with wood blocks on the inner circle blue squares: 7.831×10^{-4} *kg-m^2.*

Physical pendulum with wood blocks on the outer circle red squares: 11.18×10^{-4} *kg-m^2.*

Students should find that when the wood blocks are at the larger radius, then the moment of inertia about the pivot point axis is larger.

***Challenge #2*—Derive an equation for the *theoretical* value for the moment of inertia *about the center of mass*. Be sure to state any assumptions you are making. Then, using this equation, determine the theoretical value in kg-m^2.**

The author assumed that the mass of each of the blocks was concentrated at the center of each of the blocks. In other words, they were considered as point masses at a known radius from the center of the disk. Although one could take into account that each of the blocks is actually a cube, the derivation becomes significantly more tedious, and the resulting value is not much different.

Figure 4 shows the parameters used in the author's derivation of the theoretical equation for the moment of inertia of the disk/blocks system about their center of mass CM. The symmetric arrangement of the blocks around the circle of radius *r* implies that their center of mass is the same as the disk. *R* is the radius of the disk, while *r* (lower case) is the radius of the circle containing the blocks. *d* is the distance from the center of mass CM to the location of the axis. M is the mass of the back-to-back card stock disks, while m (lower case) is the mass of a single wood block. As indicated previously, we assume that this mass *m* is concentrated the point P, the center or the block.

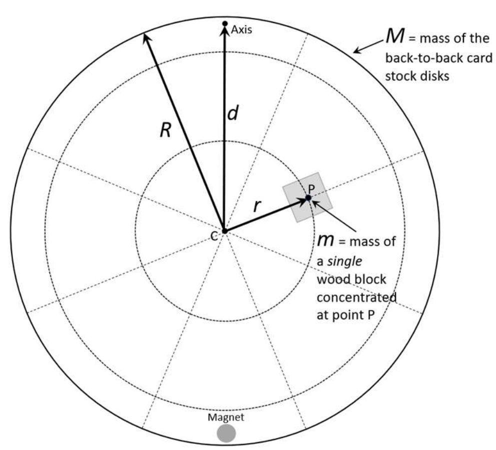


Figure 4

The moment of inertia of the system is the moment of inertia of the disk about an axis perpendicular to the center C *plus* the moments of inertia of each of the blocks about C. Tables of moments of inertia of planar figures indicate that the moment of inertia of the disk is $\frac{1}{2}MR^2$. The moment of inertia of each of the blocks about an axis through C is mr^2, under our assumption that the mass of the block is concentrated at its center. By definition of moment of inertia, the moment inertia of all 16 blocks is therefore $16mr^2$. Our final equation, then, for the moment of inertia I_{CM} of the disk/blocks system about an axis perpendicular to the disk at center of mass C is

$$I_{cm} = \frac{1}{2}MR^2 + 16mr^2.$$

Plugging in the author's values for the parameter on the right-hand side of this equation, the following results were obtained:

I_{cm} with wood blocks on the inner circle blue squares: *1.432×10^{-4} kg-m^2.*

I_{cm} with wood blocks on the outer circle red squares: *4.493×10^{-4} kg-m^2.*

***Challenge #3*—Derive an equation for the theoretical value for the moment of inertia about the axis that is very close to the top of the physical pendulums.**

Here, students need to make use of the *Parallel-axis Theorem*: If I_{cm} is the moment of inertia about the center of mass, m is the mass of the object, and w is the distance between the center of mass and an axis parallel to that at the center of mass, then

$$I = I_{cm} + mw^2.$$

In this equation m is the mass of the entire system, *i.e.*, disk plus 16 blocks. w is the distance d in Figure 4. With values for all parameters as measured by the author:

I with wood blocks on the inner circle blue squares: 7.41×10^{-4} kg-m^2.

I with wood blocks on the outer circle red squares: 10.567×10^{-4} kg-m^2.

***Challenge #4*—Determine the percent difference between the experimental and theoretical moments of inertia about the pivot axis. Comparisons should be expressed as the percent difference between your experimental and theoretical moment of inertia about the axes near the edges, based upon a fraction of the theoretical moment of inertia. The percent difference would be negative in the event that your experimental result is less than the theoretical, and positive if the experimental result if greater than the theoretical.**

The table of Figure 5 summarizes the results obtained by the author. The table shows that the author had consistent results with differences somewhat less than 6% between experimental and theoretical moments of inertia. For both physical pendulums, the experimental value was slightly higher than the theoretical value.

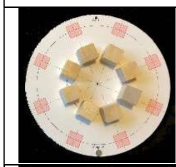
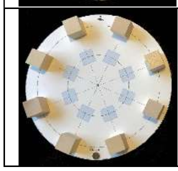
Physical Pendulum	Experimental Moment of Inertia *(kg-m^2)* $I = \frac{T^2 M g d}{4\pi^2}$	Theoretical Moment of Inertia *(kg-m^2)*	% Difference (Expt – Theory)/Theory x 100%
	7.831×10^{-4} kg-m^2	7.41×10^{-4} kg-m^2	+5.7%
	11.18×10^{-4} kg-m^2	10.567×10^{-4} kg-m^2	+5.8%

Figure 5