Physical Pendulum: Finding Moment of Inertia

Teacher's Guide

Measuring the Period of the Physical Pendulums

Figure 1 shows a typical graph of magnetic field magnitude vs. time for any of the objects studied in this investigation. In order to obtain the best possible accuracy, students should have the data rate set to the maximum possible with PocketLab—50 points/second. They should also compute the time for several periods, and not just look at a single period. In Figure 1, the time for 17 periods was noted as 19.00 seconds, giving a period T = 1.118 s

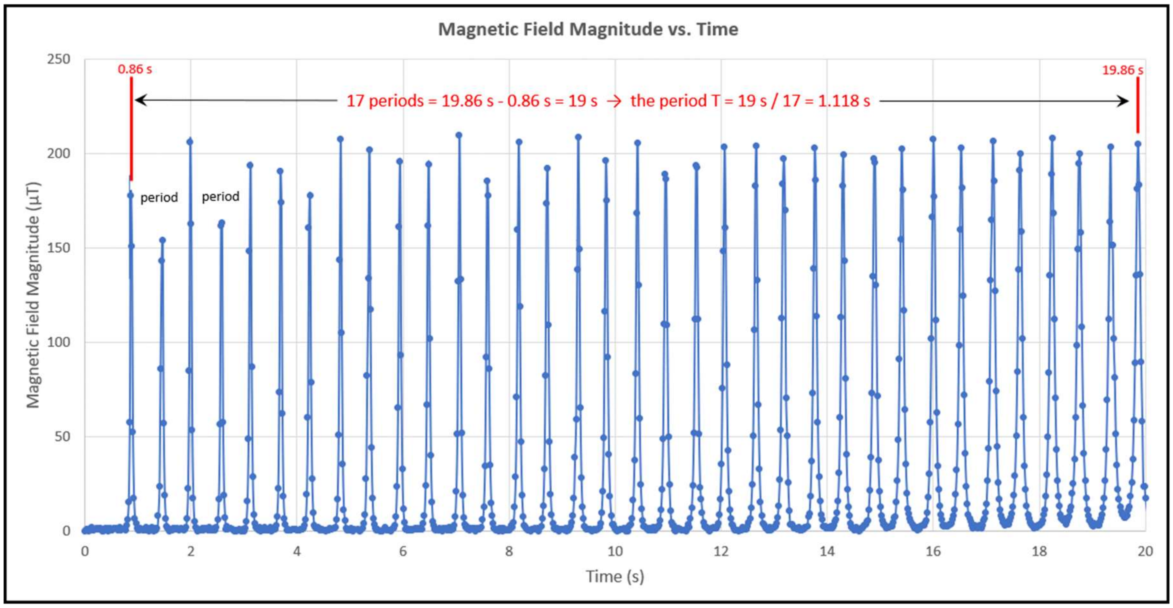

Figure 1

Square Investigation

What is the value of *n* in the equation for a square? *4*

Obtain a formula for the moment of inertia about the center of mass (barycenter) in terms of *m* and *R*.

$$I_c = \frac{1}{3}mR^2$$

Recast the equation in terms of *m* and *a*, where *a* is the length of each side of the square.

$$I_c = \frac{1}{6}ma^2$$

Use the Parallel-axis Theorem to determine an equation for the theoretical moment of inertia about a vertex of the square in terms of *m* and *a*.

$$I = \frac{2}{3}ma^2$$

Equilateral Triangle Investigation

What is the value of *n* in the equation for your equilateral triangle? *3*

Obtain a formula for the moment of inertia about the center of mass in terms of *m* and *R*.

$$I = \frac{1}{4}mR^2$$

Recast the equation in terms of *m* and *a*, where *a* is the length of each side of the equilateral triangle.

$$I = \frac{1}{12}ma^2$$

Use the Parallel-axis Theorem to determine an equation for the theoretical moment of inertia about a vertex of the equilateral triangle in terms of *m* and *a*.

$$I = \frac{5}{12}ma^2$$

Irregular Curved Shape Investigation

Students will need to come up with a way to determine the center of mass of this irregular curved shape. Probably the easiest way, though not necessarily the most accurate, would be to find the point on the irregular shape where it could be balanced on one's finger. They could also suspend the object from two or three different points on the object and draw vertical lines down from the points of suspension. These points would intersect at the center of mass. See Figure 2 for an example.

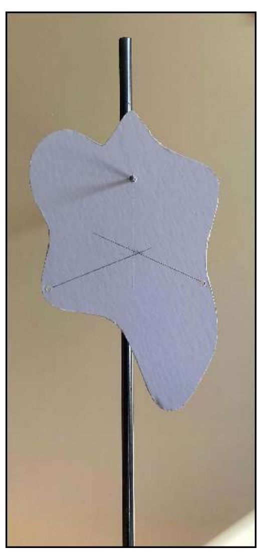

Figure 2

Object with "Negative Space" Investigation

This shape was included because the center of mass lies in the negative space, making it a little more challenging for students to determine the center of mass. The author's approach was to tape a thin piece of wrapping tissue paper into the negative space of the object. The tissue paper was selected as its mass is small compared to that of the object under study. Then, as was done for the irregular curved shape, the object was suspended from a couple of points, noting the intersection of the vertical lines from each suspension point. See Figure 3.

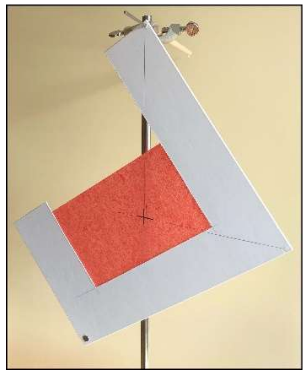

Figure 3

Example Data Table

The table below shows data collected by the author. The first four objects in the table all had percent errors with magnitudes less than 3%. For the last three objects, no theoretical values were obtained. Because of inaccuracy in locating the center of mass of these three objects, the author has less confidence in the values obtained for these three objects.

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Data Table

Object	Period T *(s)*	Mass *(kg)*	Distance between pivot and center of mass *(m)*	Experimental Moment of Inertia *(kg-m²)* $I = \frac{T^2 M g d}{4\pi^2}$	Theoretical Moment of Inertia *(kg-m²)*	% Difference (Expt – Theory)/Theory x 100%
Rectangle	0.9538	0.03637	0.149	1.225×10^{-3}	1.193×10^{-3}	+ 2.68 %
Circle	1.116	0.0745	0.203	4.68×10^{-3}	4.605×10^{-3}	+1.63 %
Square	1.0267	0.06318	0.192	3.1774×10^{-3}	3.255×10^{-3}	– 2.4 %
Equilateral Triangle	0.9229	0.04085	0.1732	1.4974×10^{-3}	1.532×10^{-3}	– 2.2 %
General Triangle	1.01143	0.04287	0.200	2.18×10^{-3}		
Irregular Curved Shape	0.7911	0.0186	0.059	1.71×10^{-4}		
Object with Negative Space	1.144	0.0395	0.23	2.91×10^{-3}		