Hysteresis with Rubber Bands

Teacher's Guide

There are a number of things that teachers can do to promote a successful rubber band hysteresis lab for their students. Here are a few, based upon the author's work in preparing this lesson:

1. Have a mass hanger and about six 100 g masses for each lab group.
2. Provide a package of assorted sizes of rubber bands. Some large width rubber bands are too stiff to provide a reasonable amount of stretch when adding masses of 100 g each. Masses of less than 50 g each (*i.e.*, 5 g, 10 g, and 20g) will likely not provide a reasonable amount of stretch for each mass that is added. Students can experiment a little to find the best rubber band size.
3. Make sure that the rubber bands used are new and unstretched. The students should not play with the rubber bands in advance of the experiment by stretching them with their hands and fingers.
4. A total stretch (after adding six masses of 100 g each) of 20 cm to 25 cm is desirable.
5. The rubber bands should be stretched as much as possible while adding masses, but not to the breaking point.
6. Masses should be added carefully, not allowing the weights to oscillate up and down.
7. The author found that after adding each mass, about 10 seconds worked well to allow the rangefinder to collect data.
8. ***With the possibility of rubber bands suddenly breaking and flying about, students should wear safety goggles.***
9. Student groups will need access to spreadsheets and other graphical analysis software such as CloudLab. Such software will serve as an analysis base for the rangefinder *csv* file that is created by the PocketLab app. The analysis will test their spreadsheet skills and creativity.

Raw Data from Voyager's Rangefinder

Figure 1 shows an Excel graph of data collected by the author. Each of the steps in the graph has been labeled with the amount of the loaded mass in kg and the average rangefinder reading in meters. At the simplest level, students can visually eye each of the steps to obtain an average. They could also let the Excel spreadsheet compute the average for each of the steps by highlighting the corresponding data in the accompanying table from the PocketLab app. Alternatively, if a student group has elected to collect the data using CloudLab, then the data analysis capability of CloudLab would allow quick computation of the step averages. Figure 2 shows how this is done in CloudLab. Data is highlighted for the desired step, as shown by the shaded region in the top half of Figure 2. CloudLab then zooms in on that region, as shown in the bottom half of Figure 2. The "Data Analysis" is then selected, and a pop-up window tells us that the average rangefinder reading in the highlighted region is 0.429 meter.

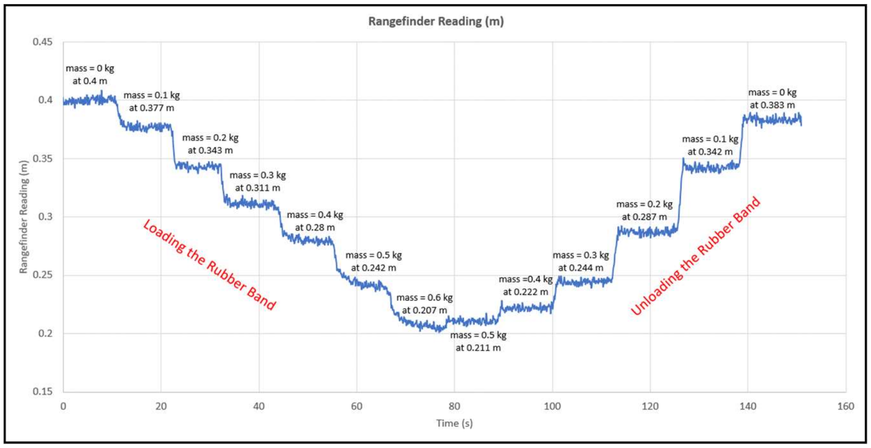


Figure 1

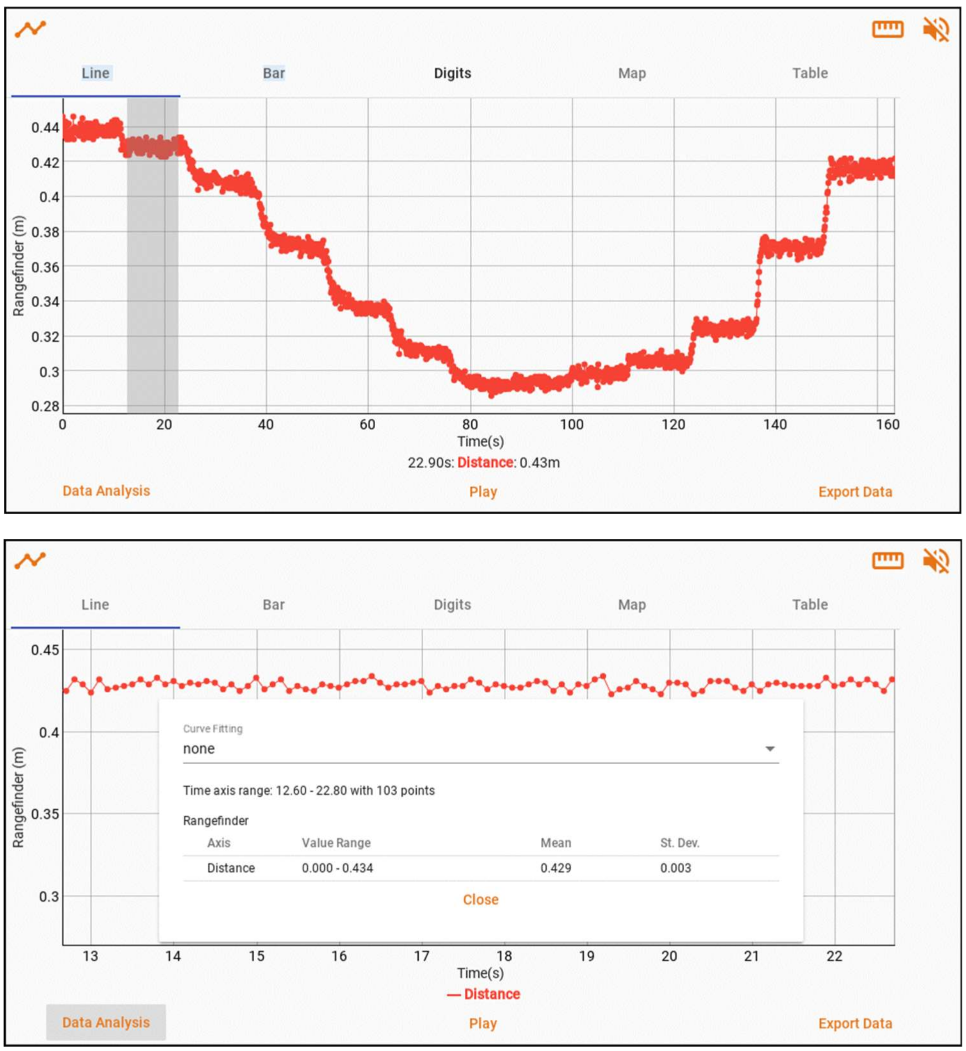


Figure 2

Spreadsheet Analysis

Figure 3 shows an Excel spreadsheet designed by the author to aid in the analysis of the raw data from the rangefinder. Formulas are shown on the left and resulting values on the right side of Figure 3.

Throughout the analysis, students must keep in mind that their *major goal is to determine the hysteresis loss* ***in Joules****.* Joules is an MKS (Meter, Kilogram, Second) measure of energy. We therefore expressed the mass in kilograms rather than grams in column B. If the students kept the rangefinder default unit of meters, then their rangefinder readings (column C) will already be in MKS meters. Since the rangefinder readings are distances from Voyager to the white cardboard at the bottom of the ring stand, the actual rubber band *stretch* values are obtained by subtracting each rangefinder reading from the rangefinder reading when the mass is zero. These calculations are shown in column D. Finally, we need to convert the masses to forces in Newtons. This is done by multiplying each mass value by 9.81 m/s^2. This is shown in column E. With this spreadsheet design we can then construct any graphs that are necessary or helpful in obtaining our ultimate goal.

	A	B	C	D	E
1		Mass (kg)	Rangefinder Reading (m)	Stretch (m)	Force (N)
2	Loading the Rubber Band	0	0.399	=$C$2-C2	=B2*9.81
3		0.1	0.377	=$C$2-C3	=B3*9.81
4		0.2	0.343	=$C$2-C4	=B4*9.81
5		0.3	0.311	=$C$2-C5	=B5*9.81
6		0.4	0.28	=$C$2-C6	=B6*9.81
7		0.5	0.242	=$C$2-C7	=B7*9.81
8		0.6	0.207	=$C$2-C8	=B8*9.81
9	Unloadin g the Rubber Band	0.5	0.211	=$C$2-C9	=B9*9.81
10		0.4	0.222	=$C$2-C10	=B10*9.81
11		0.3	0.244	=$C$2-C11	=B11*9.81
12		0.2	0.287	=$C$2-C12	=B12*9.81
13		0.1	0.342	=$C$2-C13	=B13*9.81
14		0	0.383	=$C$2-C14	=B14*9.81

	A	B	C	D	E
1		Mass (kg)	Rangefinder Reading (m)	Stretch (m)	Force (N)
2	Loading the Rubber Band	0	0.399	0	0
3		0.1	0.377	0.022	0.981
4		0.2	0.343	0.056	1.962
5		0.3	0.311	0.088	2.943
6		0.4	0.28	0.119	3.924
7		0.5	0.242	0.157	4.905
8		0.6	0.207	0.192	5.886
9	Unloading the Rubber Band	0.5	0.211	0.188	4.905
10		0.4	0.222	0.177	3.924
11		0.3	0.244	0.155	2.943
12		0.2	0.287	0.112	1.962
13		0.1	0.342	0.057	0.981
14		0	0.383	0.016	0

Figure 3

The Hysteresis Loop

In order to see our hysteresis loop, we graph stretch vs. force from the data in Figure 3. The resulting graph is shown in Figure 4. The first thing we notice is that the relationship between stretch and force is non-linear. The rubber band does not obey Hooke's Law. But most importantly, more force was required during loading than unloading. This indicates that the system has lost energy. This energy is represented by the area of our hysteresis loop—the area between the loading and unloading portions of the graph.

Where has the lost energy gone? It has been dissipated as heat. To convince students of this, have them touch a rubber band to their lip, then stretch it and again touch it to their lip. They should notice that the rubber band has heated up a little bit. When the rubber band is unstretched, the long molecules making up the rubber are all tangled up like spaghetti. The amount of disorder in the system is high. To put this in more scientific terms, the *entropy*—a measure of the disorder of a system—is high. When the rubber band is stretched, the long molecules become much more aligned. The amount of disorder in the system is lower. The entropy, i.e. the disorder, has decreased.

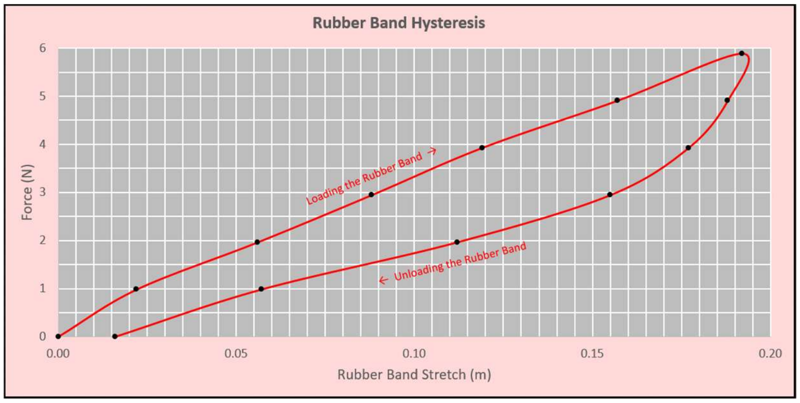


Figure 4

Determining the Heat Energy Lost in Joules

There are several ways that students can determine the heat energy in Joules that is lost for the hysteresis loop for a rubber band. *All of the methods involve finding the area of the loop*. First, the student must be convinced, or hopefully realize themselves after some thought, that the area in fact represents Joules. See Figure 5 as an aid in understanding this. Consider the area of the small blue rectangle. It is 0.5 N high, and 0.005 m wide. Its area is therefore 0.5 N * 0.005 m = 0.0025 N-m. But a N-m is the MKS measure of energy. Each little blue rectangle represents 0.0025 J of energy. Students can determine the energy lost by counting rectangles inside the loop. The author's count was about 101 blue rectangles. This means that the energy lost was 100 * 0.0025 J = 0.25 Joules.

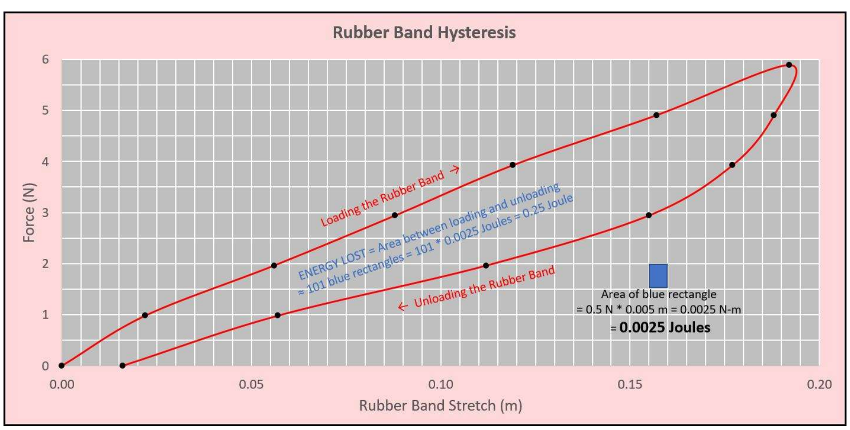
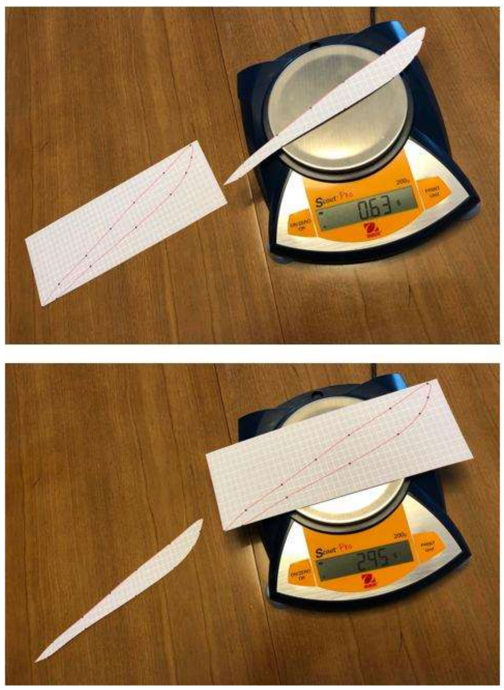


Figure 5

We now investigate three more methods that students could use to determine the heat energy lost in Joules in a rubber band hysteresis loop. Your creative students may come up with other additional methods to accomplish this goal.

The Balance Method

If your lab is equipped with a balance that measures to the nearest 0.01 g, then this is a very creative method to use. Figure 6 helps to clarify how this method works. The Force vs. Stretch graph is cut from a piece of card stock on which it has been printed. It is weighed on the balance. The hysteresis loop is then cut from the graph and it is weighed. The bottom of Figure 6 shows the simple calculations, which yield a result of 0.26 J. This method intuitively seems more accurate than counting rectangles, which is prone to error because of estimating portions of rectangles while counting.



Energy represented by the entire rectangle =
6 N * 0.2 m = 1.2 N-m = 1.2 Joules

Energy lost = Ratio of Masses * 1.2 Joules =
0.63/2.95 * 1.2 Joules = 0.26 Joules

Figure 6

Integral Software Method

There are software packages available that will automatically compute integrals (areas). One such example is the Vernier Software & Technology [Logger Pro]® software. The force and stretch data from Figure 3 are imported into Logger Pro. When applying the integral option that is available in the software, the graph shown in Figure 7 is the result. It tells us that the integral is 0.25 N-m or 0.25 Joule.

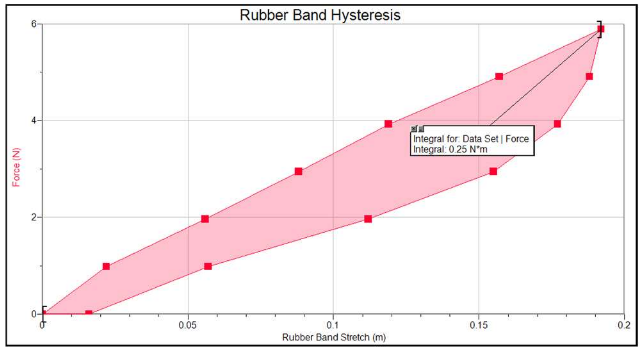


Figure 7

Web-Based Integral Software

This method is nice for students with a background in calculus. It is based upon the existence of many Web apps that can compute definite integrals. Figure 8 shows how the author prepared for use of this method. The top graph in Figure 8 shows the hysteresis loop that we saw earlier in Figure 5. The middle graph shows that we have applied an Excel linear trendline analysis to the loading portion of the hysteresis loop. A linear trendline seemed appropriate as the loading curve was visually close to a straight line. The unloading portion of the loop is more complex. The author tried several polynomial trendlines of different order. The quartic shown in the bottom graph of Figure 8 appeared to be a good fit.

With this preliminary work done, we see that we need to subtract the area under the unloading curve from the area under the loading curve in order to obtain the area of the hysteresis loop.

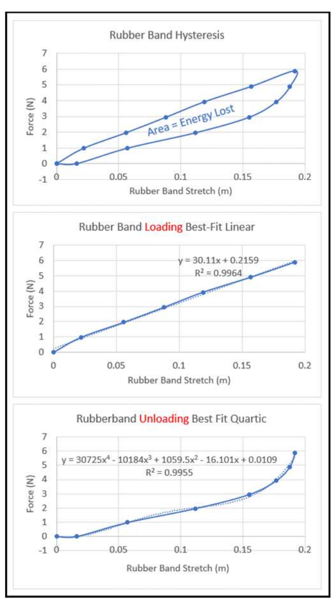


Figure 8

This would be rather tedious if we attempted to do it “by hand”, so we make use of a Web-based integral calculator called [Symbolab](), as shown in Figure 9. *x* is the independent variable *stretch*, which varies from 0 to 0.192 m, the stretch with a mass of 0.6 kg. The result is 0.248 Joule. This is in excellent agreement with the previous three methods for computing the energy lost in Joules for our rubber band hysteresis loop

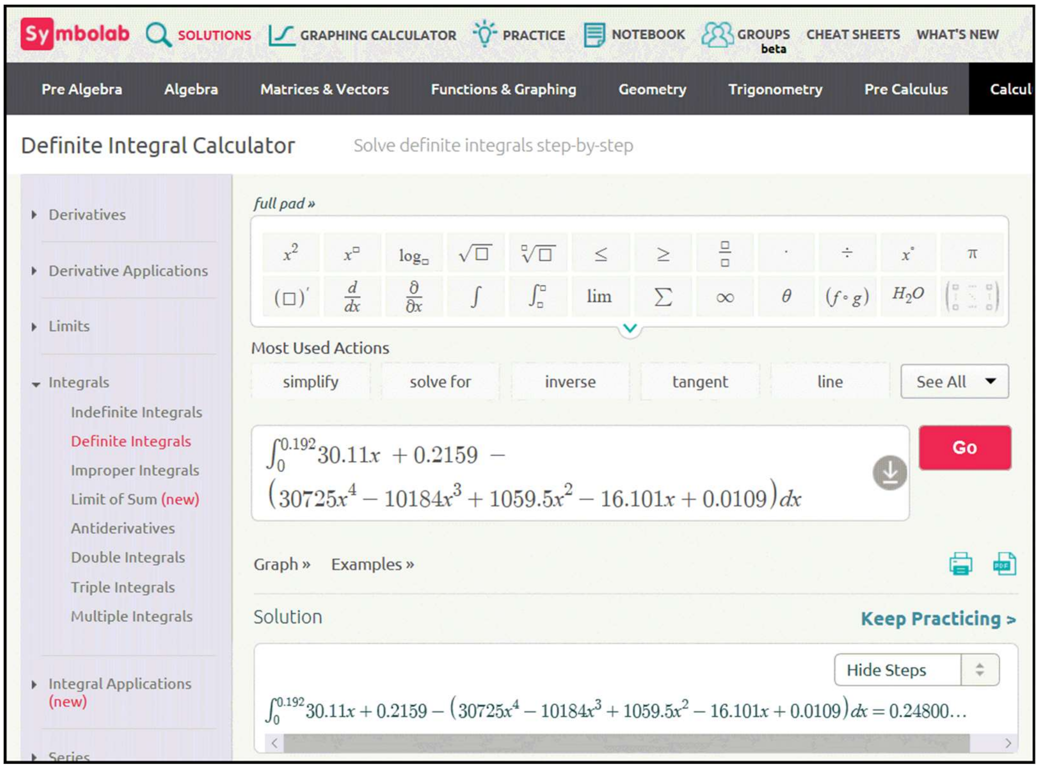


Figure 9