Periodic Motion of a Pair of Physics Carts: Experiment and Theory

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A Physics Challenge

In this lesson, AP and college students are challenged to derive equations for the periods of two fundamental modes of oscillation of a ***pair*** of coupled physics carts. Derivation will involve Hooke's law, Newton's Second Law of Motion, and principles of simple harmonic motion. Theory is then compared to experimental results obtained from PocketLab Voyager rangefinder data using Phyphox software.

For additional experiments involving two carts with Phyphox and PocketLab, click this link as well as this link.

The Experiment Setup

Figure 1 shows the experiment setup, in which three springs are used. One spring connects the two carts. Two other springs connect each cart to solid ring stands on the far right and left. PocketLab Voyagers are attached to each cart with their rangefinders facing white card boards that are leaning on the ring stands. The rangefinders will keep track of cart location while the carts are oscillating. An Android tablet in the foreground shows typical experimental data displayed with Phyphox. A good source for the springs (5 N/m) can be found by clicking this link.

Figure 1

Phyphox Software

Phyphox (***phy***sical ***pho***ne e***x***periments) is a free app developed at the 2nd Institute of Physics of the RWTH Aachen University in Germany. The author of this lesson has been working with a pre-release Android version of this app that supports BLE (Bluetooth Low Energy) technology to transfer data from multiple Voyagers to the Phyphox app. It is important to understand that this capability of Phyphox may not be available to the public until the July 2018 anticipated beta release.

The experiment of this lesson is in a file named *TwoCarStudy3.phyphox* and will be made available from the author when the Phyphox beta is released. This file can then be opened in Phyphox and will appear in the *PocketLab Voyager* category of the main screen, similar to that in Figure 2.

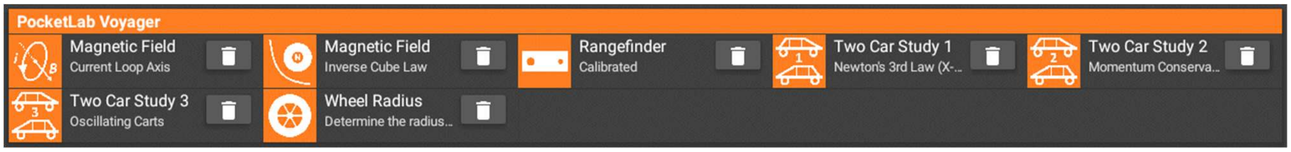

Figure 2

Performing the Experiment

The first screen that you will see after selecting the *Two Car Study 3* experiment is shown in Figure 3. The top graph indicates that PocketLab Voyager A will make use of its rangefinder. The bottom graph indicates that PocketLab Voyager B will also make use of its rangefinder. A message in the center of the screen tells you that it is scanning for Bluetooth devices with the name “PL Voyager” and asks you to pick a device. At this point you should turn on PocketLab A. “PL Voyager” will appear in the message. Click on “PL Voyager” and a message will tell you that Bluetooth is connecting to the device. You will be asked a second time to pick a device—this time turn on PocketLab B. You can now start data collection with the pulsating start triangle in the upper right corner of the screen. A 2½-minute YouTube video summarizes details related to data collection for this experiment.

It is important to remember that rangefinder Voyager A be selected *first*, and rangefinder Voyager B be selected *second*. If they are not selected in the correct order, then the two graphs will be matched to the wrong Voyagers.

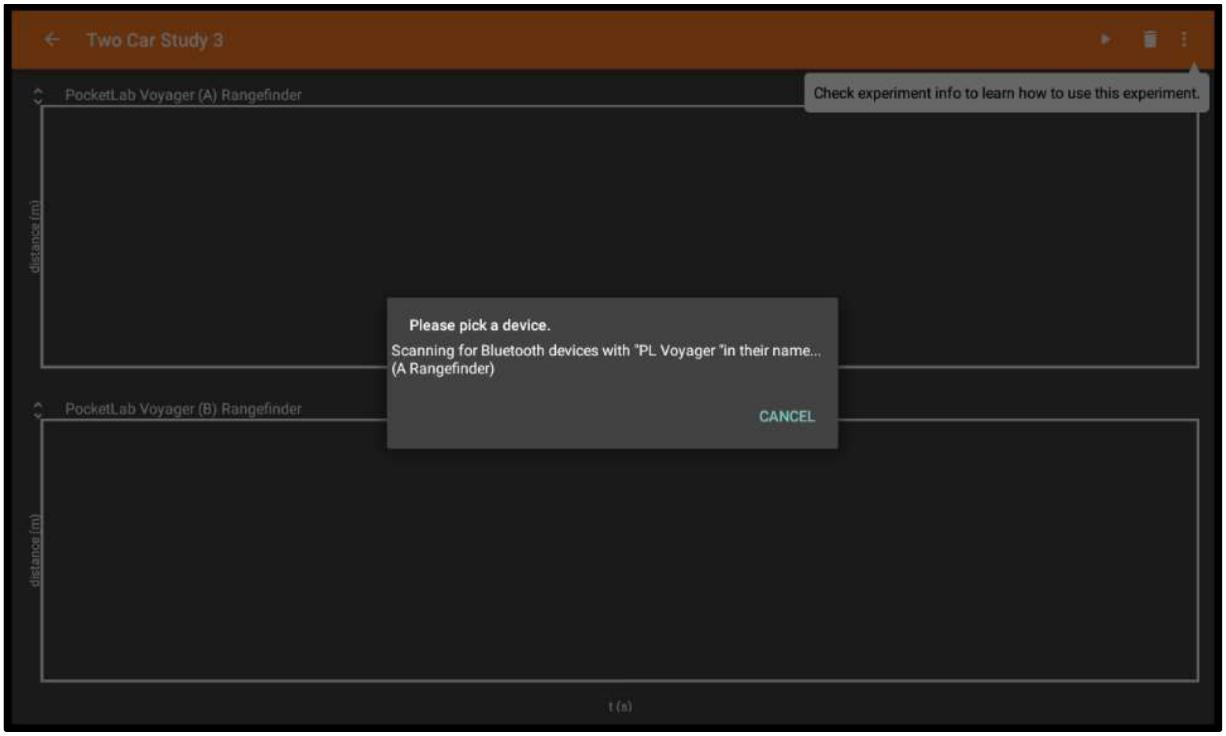

Figure 3

In order to export the data, all you need to do is click the ellipsis in the upper right corner of the screen and select *Export Data* from the drop-down menu. You can then choose the desired data format (Excel, CSV) and pick a method for sharing the data (Google Drive, Dropbox, Email, etc.) See Figure 4.

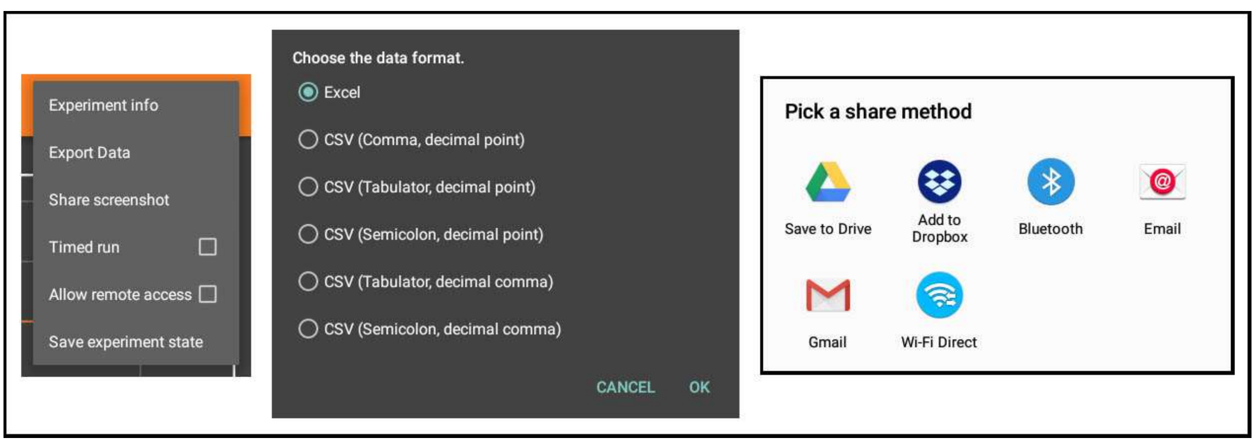

Figure 4

Analysis

Position data was collected for fundamental modes of oscillation, Mode 1 and Mode 2, as shown in Figure 5. In Mode 1, the carts are released from rest after being brought together near the center of their equilibrium positions. In Mode 2, carts are released from rest after being moved to a point near one of the ring stands, keeping their separation the same as it is when in equilibrium.

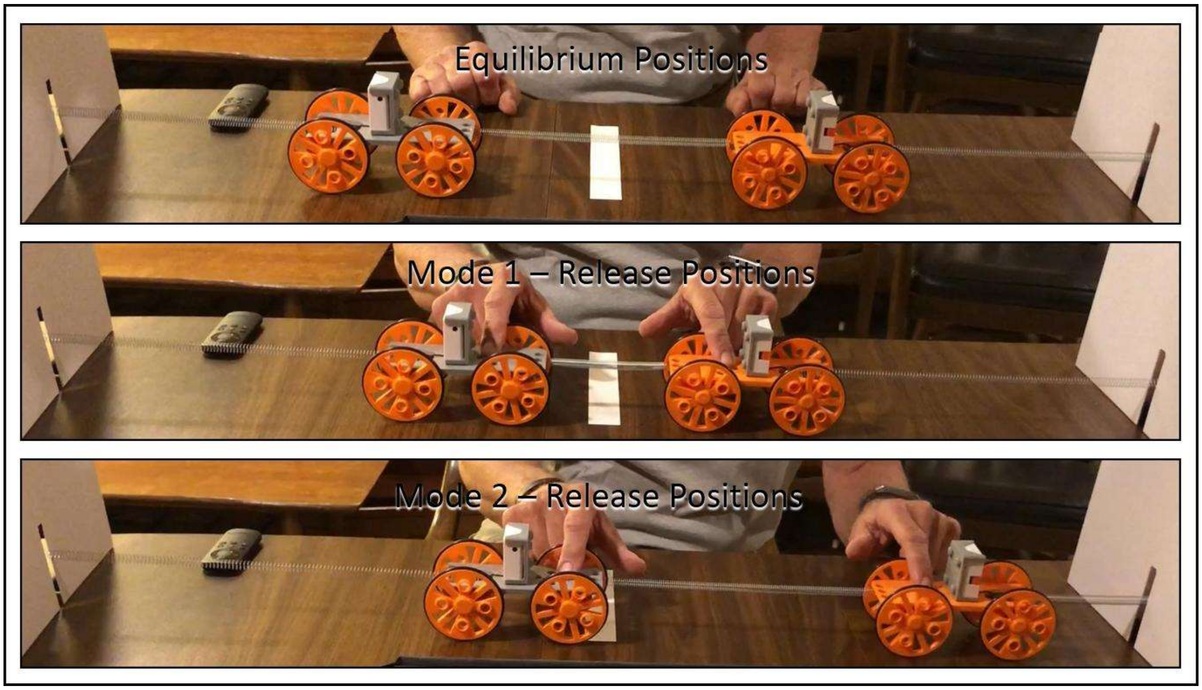

Figure 5

There are a variety of ways to approach the analysis of the rangefinder data from this experiment. The author elected to export the data from Phyphox as an Excel file. Figure 6 shows the graph that was constructed from a run of the experiment.

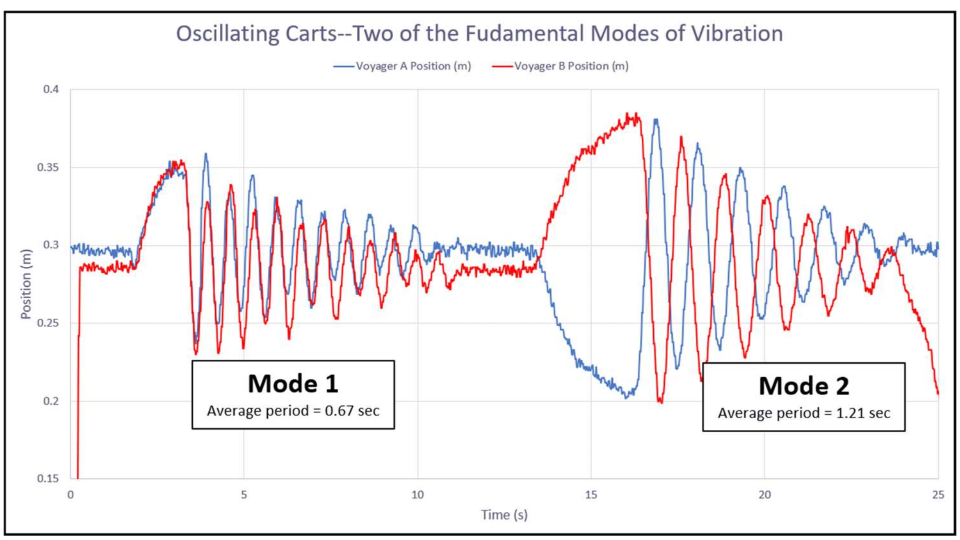

Figure 6

You need to keep in mind that the positions of each of the two carts have been measured from different cardboards from opposite directions. While we could have adjusted positions to account for this, doing so is not really necessary here. This is because we are only interested in determining the periods for the oscillations, not actual positions relative to some fixed point. Analysis of the graph reveals that the period for Mode 1 is 0.67 s, while the period for Mode 2 is 1.21 s. Also, the periods were the same for both the carts for each of the two modes of vibration studied.

Some Comments on Theory and Derivation of an Equation for the Period of Each Mode

We are not going to provide here a detailed derivation of an equation for the period of each of the two modes of oscillation. Rather, a set of procedural hints are provided as an aid for students who need some guidance on the theory. The diagram of Figure 7 may be helpful for students.

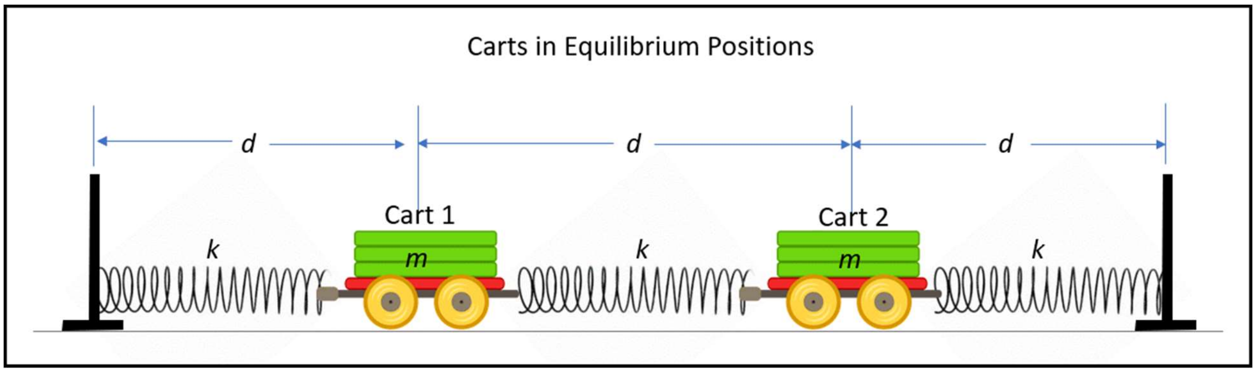

Figure 7

1. Several simplifying techniques and assumptions are helpful:
    a. The surface upon which the carts oscillate is horizontal.
    b. Assume that all three springs have identical spring constants k.
    c. Assume that the three springs are equally stretched a distance d when the system is in equilibrium.
    d. Neglect any non-conservative damping forces.
    e. Assume that each cart is a distance x from the equilibrium position when the carts are stretched to their initial mode positions (x is a positive number).
    f. Consider each of the two modes of vibration as a separate case.
    g. Assume that both carts have equal mass m.
2. Using Newton's 2nd Law as well as Hooke's Law ($F = -kx$), obtain an expression for the net force, F_{net}, on Cart 1. Do the same for Cart 2. You should be able to show that:
    a. For Mode 1, the magnitude of the net force on each cart is $3kx$.
    b. For Mode 2, the magnitude of the net force on each cart is kx.
3. Newton's 2nd Law tells us that $F_{net} = ma = m \bullet (d^2x/dt^2)$. This is a differential equation. Since the graphs of Figure 6 suggest that we have sinusoidal periodic motion, assume that this differential equation has a solution of the form $x = A \cos \omega t$, where ω is the angular velocity. Then show that:
    a. For mode 1, $\omega^2 = 3k/m$.
    b. For mode 2, $\omega^2 = k/m$.
4. Use the equation $T = 2\pi/\omega$ to determine the theoretical period T.

Final Results for the Author's Experiment

The mass of each cart plus Voyager is 0.158 kg. The nominal spring constant *k* for each of the springs is 5 N/m. The author then found the theoretical values using procedural hints 3 and 4 above:

1. For Mode 1, the theoretical period is 0.645 s.
2. For Mode 2, the theoretical period is 1.12 s.

The table of Figure 8 compares experiment and theoretical periods and shows the percentage error:

	Mode 1	**Mode 2**
Experimental Period	0.67 s	1.21 s
Theoretical Period	0.645 s	1.12 s
%-Error	3.9%	7.1%

Figure 8

Student discussion could center on possible explanations for the percentage errors when comparing experiment and theory. Did students measure the spring constant *k* or use the nominal value provided by the vendor? Does averaging the values from several student groups result in less percentage error?