**A Study of a Sail Cart Confined to a Track:**
**Acceleration as a Function of Sail Angle**

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***Introduction***

This lesson is motivated by an article by Paul G. Hewitt entitled "Sailing into the Wind: A Vector Explanation", appearing in the *Summer 2017* edition of NSTA's ***The Science Teacher***. Why not put a sail on a Teacher Geek® cart powered by wind from a fan and confined to move along a track, as shown in Figure 1? Then have the students quantitatively investigate the functional relationship between the acceleration of the cart and the angle of the sail, both in theory and empirically from data collected from the PocketLab *VelocityLab* app.

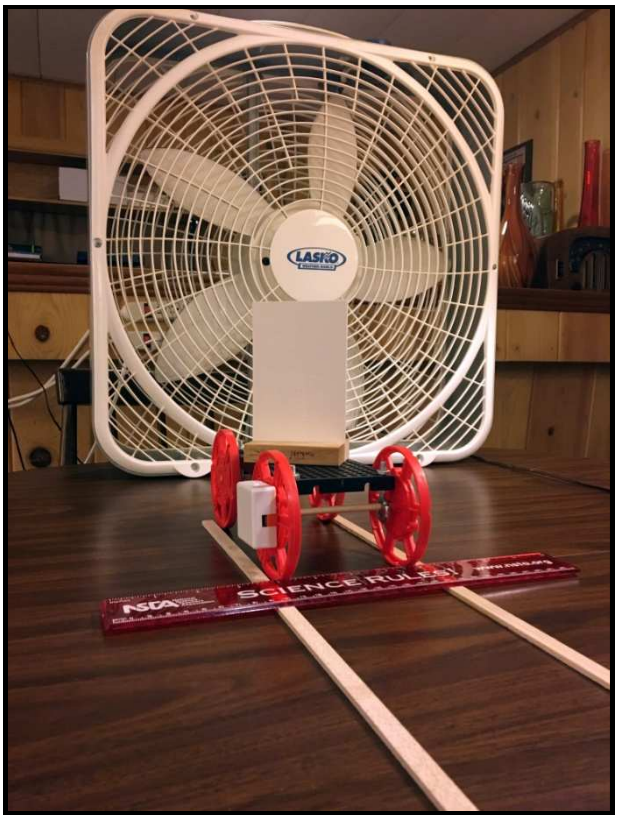


*Figure 1*

The sail is a small piece of stiff cardboard, sandwiched between a pair of wood blocks that are taped to the cart with double sided tape. An NSTA "Science Rules" ruler keeps the cart from moving until data collection is ready. The track consists of two long pieces of balsa wood taped to the table with the double sided tape. Either PocketLab One or PocketLab Voyager can be used with a data rate setting of 10 points/second in the VelocityLab app.

***Theory***

Figure 2 shows a bird's eye view of the cart along with a free body diagram of the forces involved. The force of gravity is balanced by the normal force of the wheels. These are not shown in the diagram. In our analysis, frictional forces such as those involved with the cart's axles and rolling friction are ignored. The force of the wind $F_w$ on the sail is *not* in the direction of the wind—rather, it is approximately perpendicular to the surface of our flat sail. This comes from a basic principle of fluid dynamics which tells us that for gases or liquids, their force is perpendicular to the surface on which they impinge. This force is shown by the blue vector in Figure 2, and can be thought of as the sum of two forces shown by the red vectors, one perpendicular to the track $F_{\perp}$, and one parallel to the track $F_{\parallel}$. The perpendicular component is balanced by the force of the track on the cart $F_T$. This keeps the cart confined to motion along the track in much the same way that a boat's keel keeps the boat moving forward.

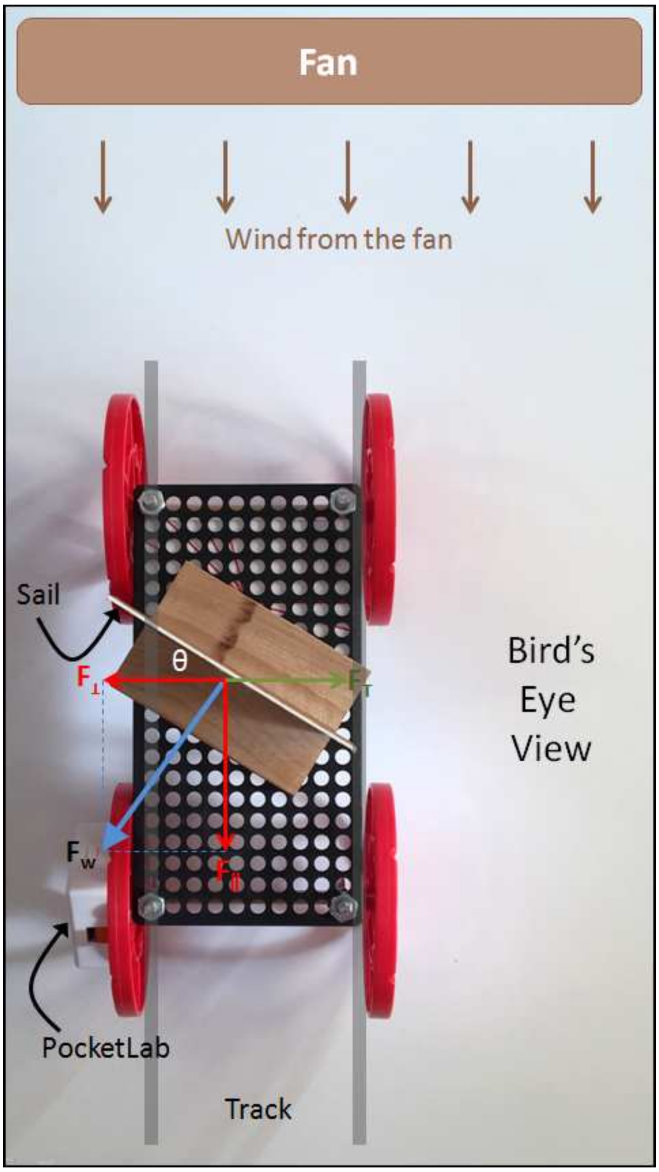


*Figure 2*

The acceleration of the cart is in part caused by the component of the wind's force that is parallel to the motion of the cart. If θ is the angle that the sail makes with $F_{\perp}$, then $F_{\parallel}$ is proportional to $F_W \cos\theta$. In other words, the force and acceleration of the cart are proportional to cos θ.

But the acceleration is ***also*** affected by the area of the sail that is intercepted by the wind. If *A* is the area of the sail, then *A* cos θ is the area intercepted by the wind. As θ increases from 0 to 90°, this area decreases from *A* to zero. Therefore, the force and acceleration are *also* proportional to cos θ *for this reason*. We conclude that $F_{\parallel}$, which is also the net force on the cart, as well as the acceleration, are proportional to $\cos^2\theta$.

Figure 3 shows graphs of cos θ and $\cos^2\theta$ versus θ, with θ varying from 0 to 90°. These are the angles through which we will vary θ during our empirical investigation. We would therefore expect a graph of acceleration versus angle to take on the shape shown for the red curve, $\cos^2\theta$.

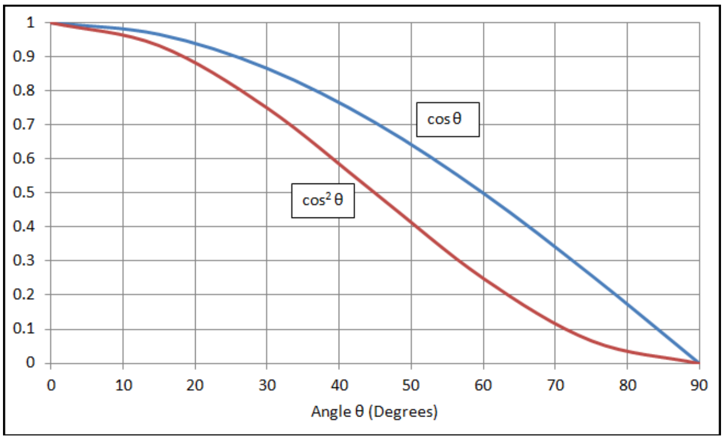


*Figure 3*

Figure 4 shows a close-up of the cart bed. A piece of paper has been taped to the underneath side of the cart with red lines drawn 15° apart. This makes it easy to set the sail to specific angles of 0°, 15°, 30°, 45°, 60°, 75°, and 90°. Thus, there will be seven runs for collecting VelocityLab data. (Figure 4 shows the sail at an angle of 30°.) A *pdf* file containing this 15° grid is included with this lesson. It can be printed and cut to fit the cart.

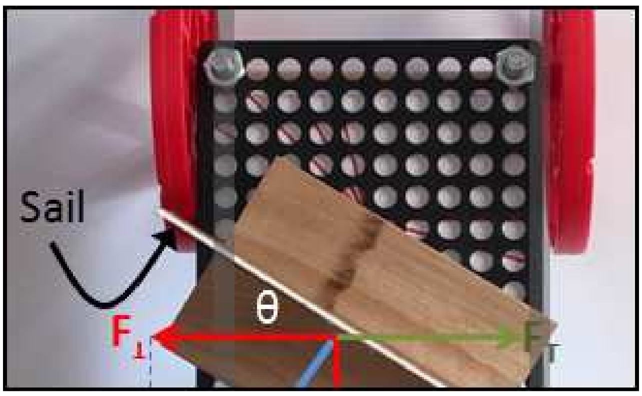


*Figure 4*

***Analysis***

Figure 5 shows an Excel graph of data collected by the VelocityLab app for θ = 0°, when the sail is perpendicular to the direction of the wind, *i.e.*, facing the wind directly. Graphs at other angles are similar in nature to this graph. The cart was released at about 1 second, had a rather uniform acceleration from 1 to 2 seconds, and hit the stopping bumper at about 2.7 seconds.

The acceleration can be obtained in three different ways:

1. Apply a quadratic fit $y = Ax^2 + Bx + C$ to the position graph for the region between 1 and 2 seconds, where the acceleration appears to be most constant. The acceleration would be given in $m/s^2$ by the second derivative, whose value is $2A$.
2. Apply a linear fit $y = mx + b$ to the velocity graph for the region between 1 and 2 seconds, where the velocity is close to a straight line and the acceleration appears to be most constant. The acceleration would be given in $m/s^2$ by the slope $m$ of the straight line of best fit.
3. Average the values on the acceleration graph between 1 and 2 seconds, during which the acceleration appears to be reasonably steady. The acceleration will be in g's.

In all three cases we get a value for the acceleration of 0.44 $m/s^2$ (or 0.044 g).

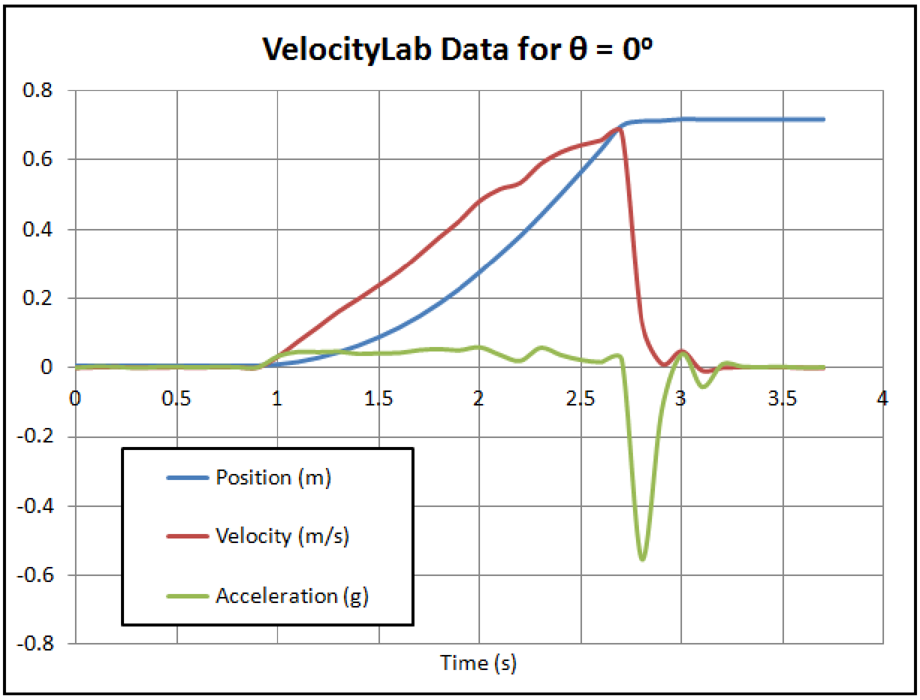


*Figure 5*

Figure 6 summarizes the results of our experiment. The graph was obtained from Vernier Software & Technology's Logger *Pro*® software by applying a fit to a general $\cos^2$ curve. ***With a correlation of 0.9660, we have strong support for our discussion in the "Theory" section of this document that acceleration is proportional to $\cos^2$ θ.*** Any software that provides ample curve fitting choices or the ability to define custom curves for fitting could be used.

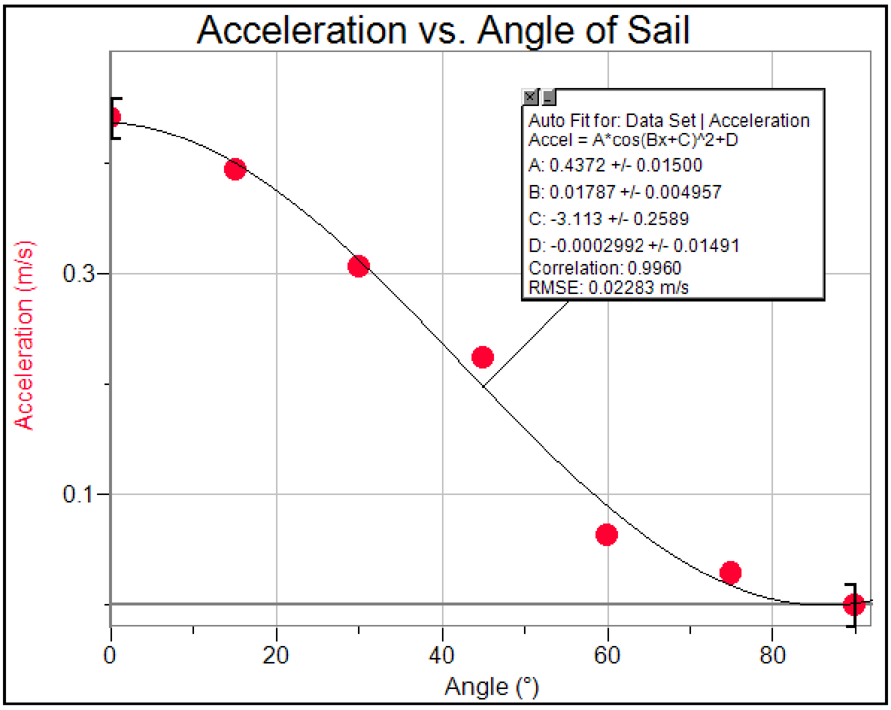
Acceleration vs. Angle of Sail
Auto Fit for: Data Set | Acceleration
Accel = A*cos(Bx+C)^2+D
A: 0.4372 +/- 0.01500
B: 0.01787 +/- 0.004957
C: -3.113 +/- 0.2589
D: -0.0002992 +/- 0.01491
Correlation: 0.9960
RMSE: 0.02283 m/s
Acceleration (m/s)
0.3
0.1
0
20
40
60
80
Angle (°)

*Figure 6*