VelocityLab/Voyager:
An Experiment in Energy and Momentum Conservation

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Introduction

Figure 1 shows the experiment setup for this lesson. A cart of mass *M* is at rest on a horizontal surface. A sphere of mass *m* is attached to a piece of string that pivots about a fixed point P. The sphere is initially at rest at location 1. While the string is horizontal, the sphere is released and swings down to position 2, where it collides elastically with the cart. The distance from point P to the center of the sphere is *h*. Assume that the mass of the string is negligible compared to that of the sphere, and that the friction in the wheel bearings, as well as the friction between the wheels and the horizontal surface, is negligible. Your tasks are:

1. To determine the *theoretical speed* of the cart immediately after impact from the sphere, by considering the laws of conservation of energy and conservation of momentum.
2. With Voyager mounted to the cart, use data from the VelocityLab app to determine the *actual speed* of the cart immediately after impact from the sphere.
3. Compute the percent difference between the theoretical and actual speeds, and discuss possible reasons for the difference.
4. Compute the average force during the impact of the sphere with the cart.

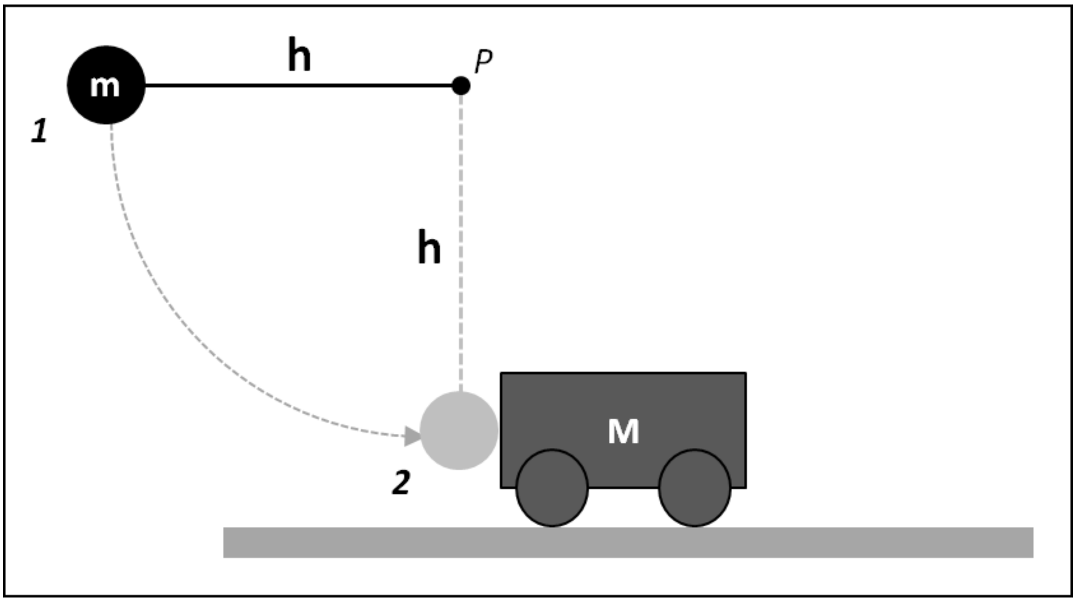

Figure 1

Determining the Theoretical Speed of the Cart

While the sphere falls, gravitational potential energy is converted to kinetic energy. The initial gravitational potential energy is *mgh*. The kinetic energy of the sphere just before impact with the cart is $\frac{1}{2}mv_o^2$, where v_o is the speed just before impact. If we assume that there is negligible air resistance and that the pivot point P is frictionless, the law of conservation of energy tells us that $mgh = \frac{1}{2}mv_o^2$. From this we can conclude that the speed of the sphere just before impact with the cart is $v_o = \sqrt{2gh}$.

If we let *v* be the speed of the *sphere* just after impact and *w* be the speed of the *cart* just after impact, then conservation of energy tells us that

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}Mw^2. \quad \text{[Equation 1]}$$

Conservation of momentum tells us that

$$mv_o = mv + Mw. \quad \text{[Equation 2]}$$

With *m, M, g, h,* and v_o all quantities with known values, the result is a pair of simultaneous equations in two unknowns *v* and *w*. We are specifically interested in the value of *w*, the speed of the cart immediately after impact. Since Equation 1 is not linear, solving this pair of equations algebraically gets rather tedious and could easily lead to errors if one is not ***extremely*** careful with the algebra and use of the quadratic formula.

An alternative approach to the solution is to use a simultaneous equation solver available on the internet. The author made use of WolframAlpha's simultaneous equations solver*, which can be used with a maximum of two variables or unknowns. First, the values for the known quantities m = 0.088 kg, M = 0.546 kg, g = 9.81 m/s^2, h = 0.42 m, and v_o = 2.87 m/s are substituted into equations 1 and 2, giving us the equation pair:

$$0.363 = 0.044\, v^2 + 0.273\, w^2$$
$$0.253 = 0.088\, v + 0.546\, w$$

Figure 2 shows a screen print of the equation solver's solution. There are two solutions. The first is the correct answer with the cart moving forward at a speed of about 0.8 m/s, and the sphere rebounding at a speed of about 2 m/s. The second solution is extraneous and cannot happen, as it has the sphere and cart moving forward with the sphere overtaking the cart!

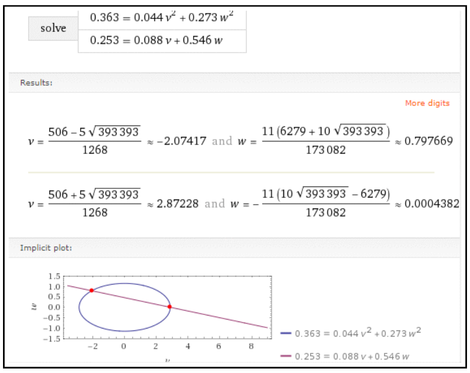

Figure 2

Determining the Actual Speed of the Cart Using Voyager/VelocityLab

Figure 3 shows the author's setup for doing the experiment. A wood sphere (from Michael's) is suspended from a ring stand via a string. The mass of the wood sphere is 0.088 kg, and the distance from the pivot point to the center of the sphere is 0.42 m. A wood block is mounted to the top of a PocketLab cart using picture hanging strips. Voyager is mounted to one of the cart wheels using double stick tape. The mass of the cart plus wood block plus Voyager is 0.546 kg.

Figure 3

Voyager works somewhat better than PocketLab 1 as Voyager can collect data at a higher rate (50 points per second), providing greater detail during the impact of the sphere on the cart's block. Position, velocity, and acceleration data are collected using the VelocityLab app. A combined data and video *mp4* file is included with this lesson. The VelocityLab csv file was imported into Excel, and the graphs of Figure 4 were then constructed.

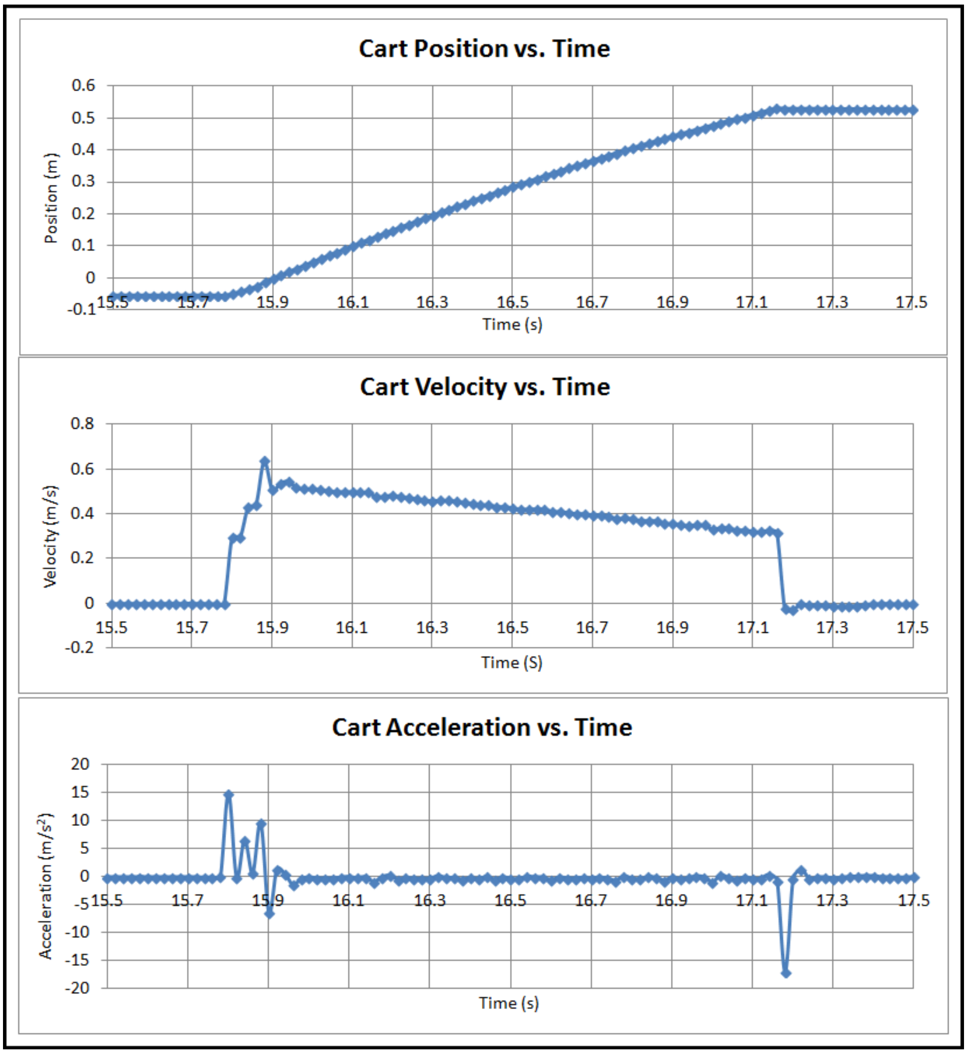

Figure 4

The velocity graph shows that the speed of the cart immediately following the impact of the sphere was about 0.6 m/s. When this is compared to the theoretical speed of 0.8 m/s from the previous section of this document, the percent difference is about 25%. We'll leave it to students to come up with reasonable explanations for the large percent difference. Reasons will likely relate to differences in the *assumptions* behind the theoretical speed versus actual experimental conditions.

Computing the force During Impact

The velocity and acceleration graphs indicate that the actual impact appeared to last for about 0.1 s, from 15.8 to 15.9 seconds. Since the impulse equals the change in momentum during the impact, *i.e.,* $F\Delta t = M\Delta v$, we should be able to approximate the force of the impact. We know that Δt is about 0.1 sec, M = 0.546 kg, and Δv is about 0.6 m/s. This implies that the average force during impact was about 3 N.

* http://www.wolframalpha.com/widgets/view.jsp?id=b42a80c01d9b3bb5bb385d4fba81a0c5