# PocketLab Voyager/LEGO®: A Study of the Half-Atwood Machine

**Richard G. Born**
**Associate Professor Emeritus**
**Northern Illinois University**

***Introduction***

A widely used experiment for studying Newton's Second Law of Motion makes use of a Half-Atwood machine. In this experiment a cart on a horizontal surface is tied to a mass hanging over a pulley. Upon releasing the hanging mass, the cart begins to accelerate. The magnitude of this acceleration is a quantity of great interest.

With cart accelerations typically between 0 and 1 m/s$^2$, and Voyager accelerometer resolution on the order of 0.1 m/s$^2$, acceleration data is quite noisy. A much better method is to collect position data for the cart using Voyager's IR range finder. A quadratic fit $x = At^2 + Bt + C$ can then be applied to the resulting position vs. time graph. Since velocity $v = dx/dt$, we have $v = 2At + B$. Since $a = d^2x/dt^2$, we have $a = 2A$. So, twice the value of $A$ in our quadratic fit gives the acceleration!

In this lesson, a Half-Atwood machine, as shown in Figure 1, is constructed using parts from LEGO®'s Simple & Powered Machines Set. Voyager is mounted (using picture hanging strips) to the back of the cart with its IR sensor facing a large piece of white foam board (not shown in the picture). The pulley holder is mounted to the table using picture hanging strips. A small strip of balsa wood is used to quickly release the cart when ready for data collection. For purposes of the picture in Figure 1, the cart and pulley are shown rather close together. Actually, the cart is released when it is about 65 cm from the pulley. Voyager is set to collect data at a rate of 50 points per second.

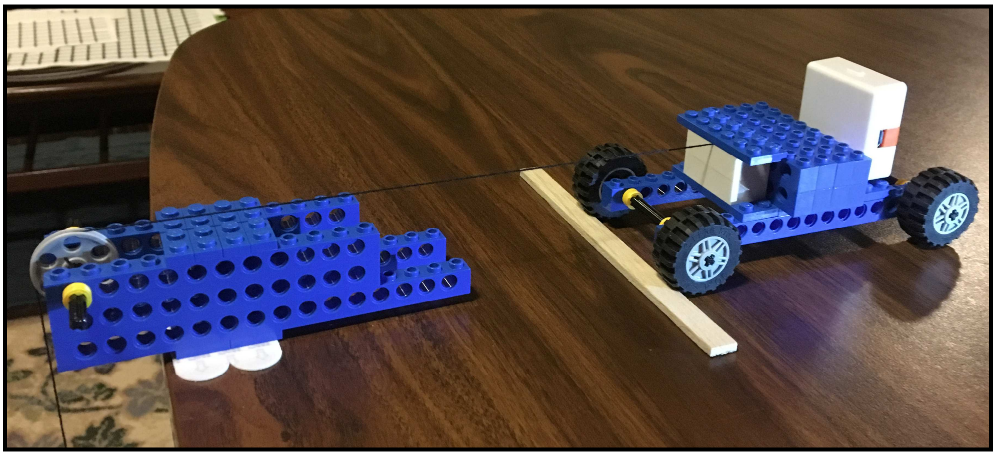

*Figure 1*

Figure 2 shows the entire Half-Atwood experiment setup. The cart is ready to be released in front of the poster board. The string that is connected to the cart passes over the pulley. The mass, hanging from the string, consists of a variety of LEGO® pieces. There will be more to say about this mass after we study the dynamics and free body diagrams.

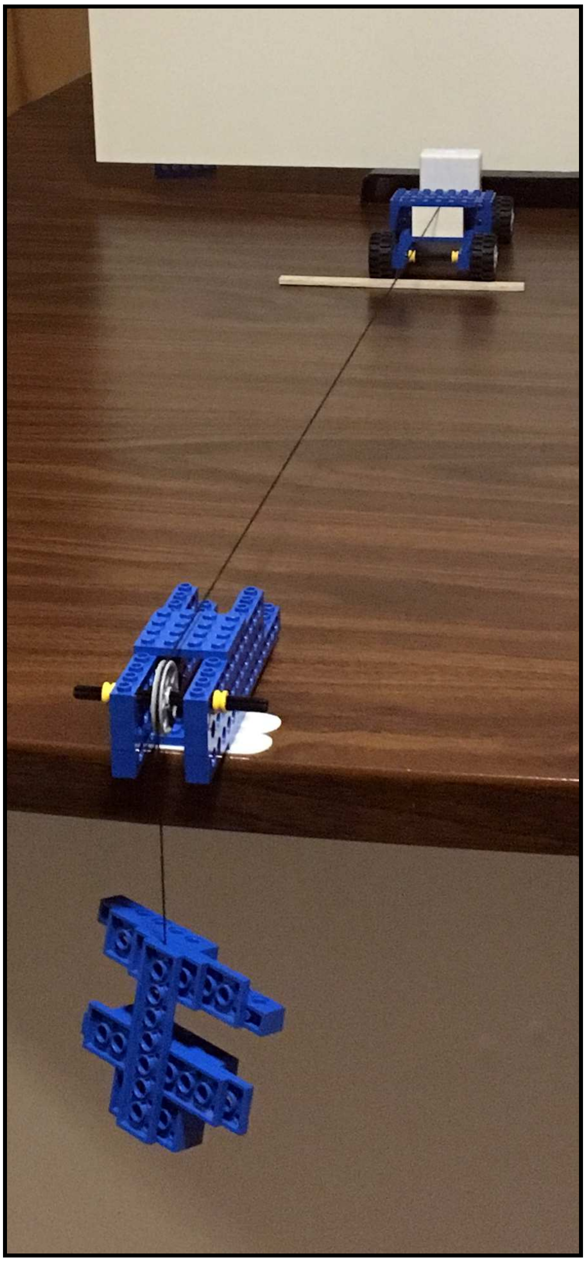

*Figure 2*

***Half-Atwood Machine Dynamics***

Figure 3 shows the free body diagrams and equations for an ideal (frictionless) Half-Atwood machine. We assume that the hanging mass m is accelerating downward, *i.e.,* mg > T. Therefore, the net force on m is ma = mg – T, as shown in equation 2. The string is taut and "transfers" the tension to the cart. Since the cart, confined to move on the table, has no vertical motion, the normal force N balances the gravitational force Mg. Therefore, the net force on M is Ma = T, as shown in equation 1. A little bit of algebra yields the important "blue-star" equation.

This “blue-star” equation provides us with the beauty of the Half-Atwood machine. The acceleration of gravity is constant. The equation therefore tells us that the acceleration of the cart (or the hanging mass) is proportional to the hanging mass m *if we keep the sum of the masses M + m constant*. How do we accomplish this? We do this **by transferring mass from the cart to the hanging mass**. Each time we transfer some mass, we note the value of the hanging mass m and use Voyager’s range finder to determine the acceleration of the cart. It is our hope that a graph of acceleration vs. mass m will be a straight line, providing good evidence that a is proportional to m.

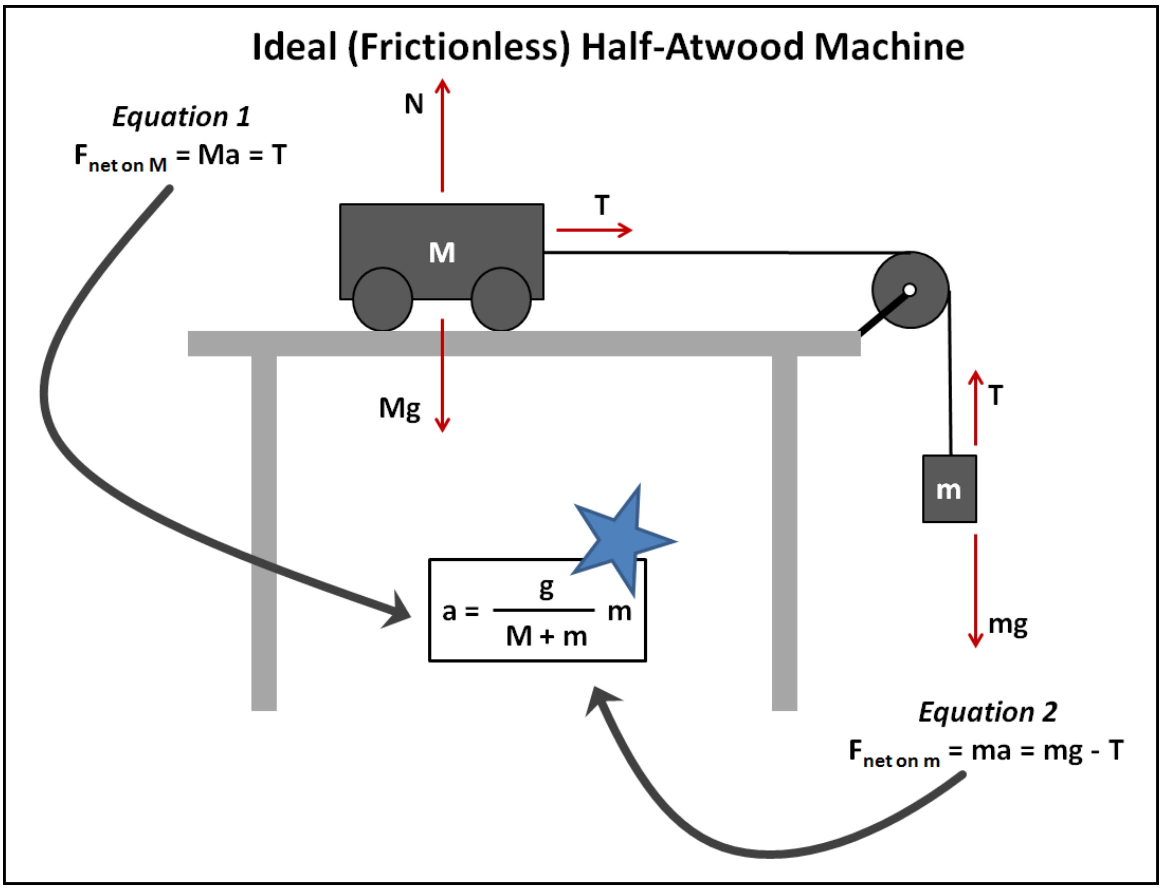


*Figure 3*

***Data Analysis***

Figure 4 contains two graphs that were constructed in Excel from Voyager range finder data obtained from the csv file created by the PocketLab app. The graphs correspond to the data run for which the hanging mass *m* was 7.23 g. The top graph shows that the cart was released at about 1.2 seconds, accelerated while the hanging mass fell, and finally stopped at about 3.2 seconds. The region shown in red was enlarged in the bottom graph for the purpose of performing a quadratic curve fit on this region of acceleration of the cart. Based upon our discussion in the introduction, we find that the acceleration is about 2 x 0.0294 = 0.185 m/s$^2$.

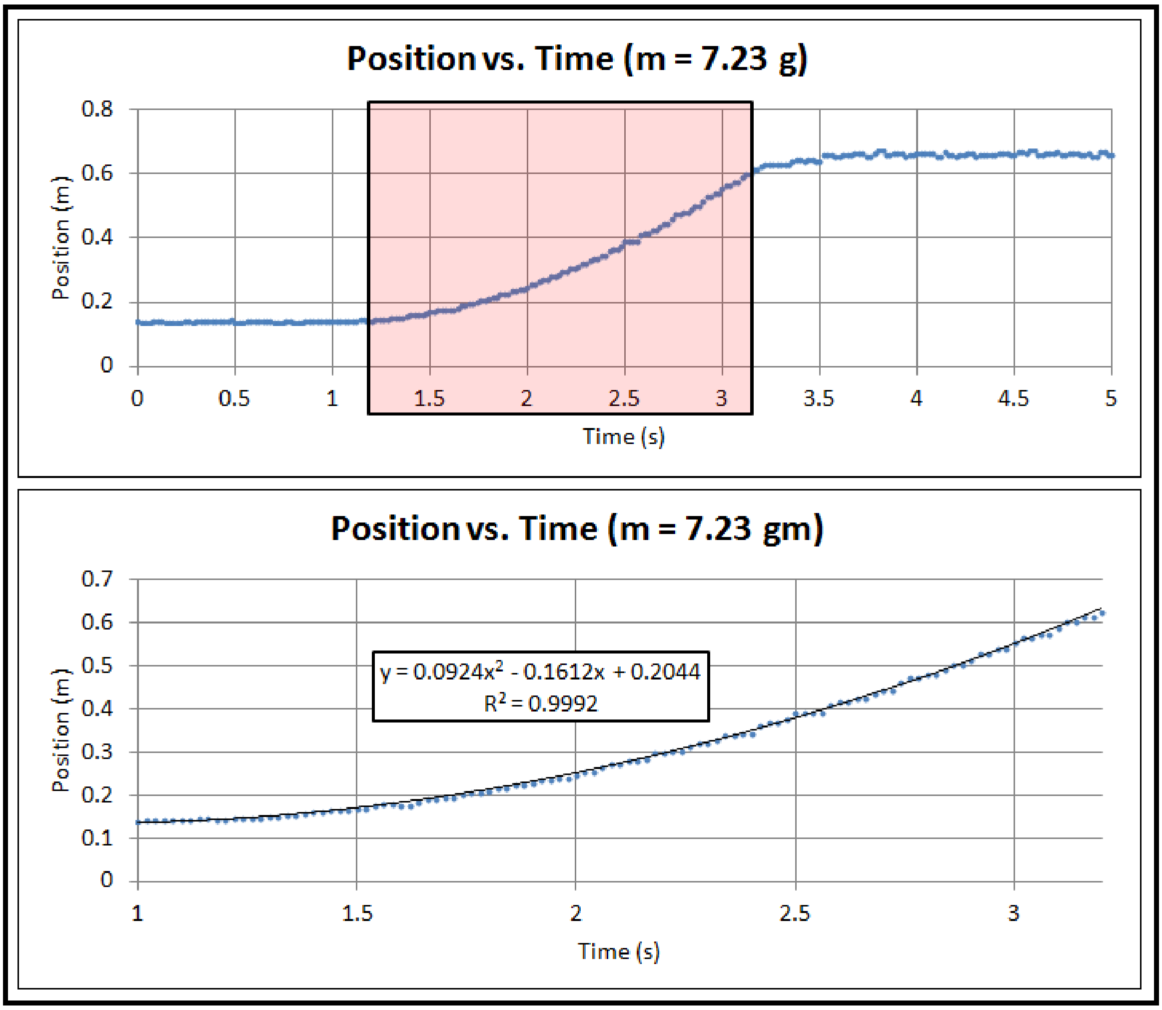


*Figure 4*

Figure 5 shows an Excel chart that summarizes the *(hanging mass, acceleration)* data pairs that were obtained by the procedure outlined in the example of Figure 4. A linear fit to the data, with an $R^2$ value of 0.9973, provides strong evidence that the acceleration is proportional to the value of the hanging mass *m* when the sum of the masses of the cart and hanging mass is kept constant. The acceleration increases by 0.0527 $m/s^2$ per gram of additional hanging mass. We note that the x-intercept of the straight line of best fit occurs at about 4 g. Although we have attempted to minimize friction by providing the wheels on the cart and using a low friction pulley, friction is never-the-less still present. A minimum of 4 g for the hanging mass is required just to overcome frictional forces involved in our Half-Atwood machine.

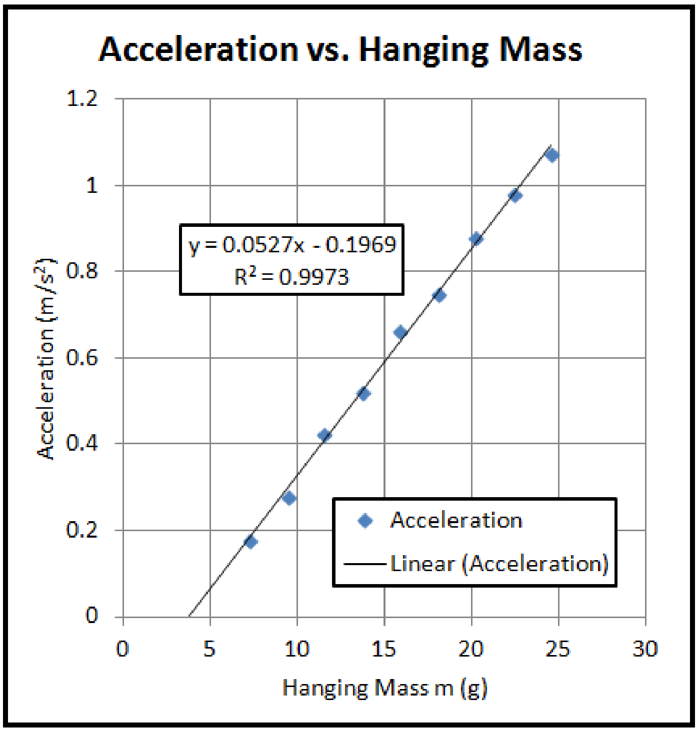
Acceleration vs. Hanging Mass
y = 0.0527x - 0.1969
R² = 0.9973
Acceleration
Linear (Acceleration)
Acceleration (m/s²)
Hanging Mass m (g)
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30

*Figure 5*