PocketLab Voyager: A Quantitative Study of Torsional Harmonic Oscillators

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Introduction

Many introductory physics courses at high school and college levels include experiments involving translational spring/mass oscillators, with a goal of determining the spring constant *k* from the period of oscillation and the suspended mass *m*. The value of *k* determined in this manner is then compared with a direct measurement in N/m by measuring the force when stretching the spring a known distance.

Springs, however, can not only be stretched—they can also be twisted. Such twisting results in what is often called a *torsional harmonic oscillator*. For small twists, the torque τ applied while twisting the spring is proportional to the angular displacement θ. This is expressed by the equation $\tau = -\kappa\theta$, with the minus sign indicating that the torque is a restoring force that is directed opposite to the angular displacement. The constant κ, called the *torsional constant*, depends on properties of the spring in the same way that the constant *k* depends on properties of the spring when considering a translational spring/mass oscillator and its well-known equation $F = -kx$. The solution to the equation of motion for a torsional oscillator is analogous to that for a translational oscillator and is shown in Figure 1.

Translational	**Torsional**	
$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{I}{\kappa}}$	T = period m = mass k = translation spring contant I = moment of inertia κ = torsional spring constant

Figure 1

Considering the case for the torsional oscillator, if the moment of inertia *I* about the axis of rotation is known and the period *T* is measured, then the torsional constant κ can be determined. Alternatively, if the torsional constant κ is known and the period *T* is measured, then the rotational moment of inertia *I* can be determined. In the experiment of this lesson, students will:

1. Determine the translational and torsional constants for a spring.
2. Determine the moment of inertia about an axis of rotation of a common geometrically shaped object and compare this value to the theoretical formula for the moment of inertia about that axis.

PocketLab Voyager is perfect for performing this experiment. Voyager is taped to the mass hanging from the spring. The mass is given both an initial vertical translation and a torsional twist and then released. While simultaneously bobbing up-and-down and twisting back-and-forth, the two motions are recorded by Voyager. The period of the translational motion is recorded by the acceleration sensor. The angular velocity sensor concurrently records information for measuring the period of the torsional oscillation. See Figure 2 for a snapshot from a video recorded by the PocketLab app. A link to the video accompanies this lesson. Since Voyager is taped to the suspended mass, Voyager's mass and moment of inertia must be taken into account in all calculations.

A perfect spring to use in this experiment is a nominal 5 N/m spring in a springs set sold by Vernier Software & Technology.

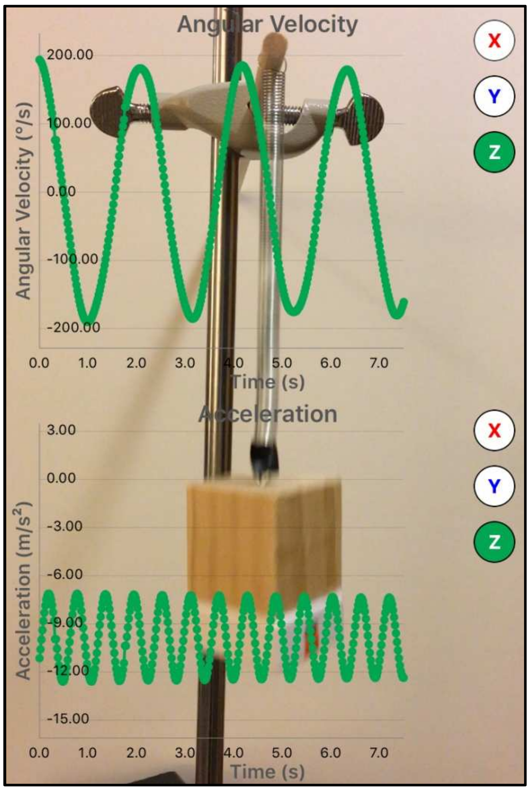

Figure 2

The spring has been attached to a wood dowel held on a ring stand by a loop at one end of the spring. The other end of the spring is attached to an eye bolt that has been screwed into a wood block. Voyager is tapped to the bottom of the wood block with the orange side down using removable poster tape. A small piece of black electrical tape is wrapped around the bottom loop of the spring and the eye bolt to keep them tightly bound and from slipping while the oscillations take place. The way in which Voyager has been mounted to the wood block means that the z-component of angular velocity is of interest in determining the period of torsional oscillation and the z-component of acceleration is of interest in determining the period of translational oscillation.

It is probably a good idea to have the students first check to see if the period is dependent on the angular displacement of the spring's twist. This can be accomplished by recording the angular velocity for several minutes, starting with a large initial twist. Students should find that with the Vernier spring, the period is independent of the angular displacement. See Figure 3. Note that due to frictional losses, angular displacement decreases as the angular velocity decreases.

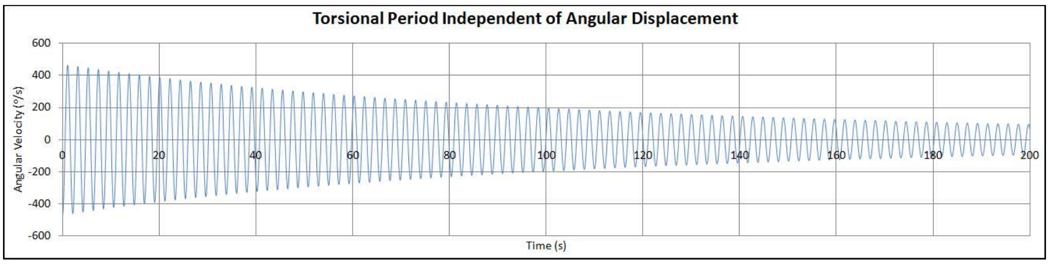

Figure 3

Experiment Results

Figure 4 contains graphs constructed in Excel of the angular velocity and acceleration data shown in the accompanying video and collected by Voyager and the PocketLab app. The graphs show that the period for the spring's torsional displacement is 2.12 s, while the period for the translational displacement is 0.58 s.

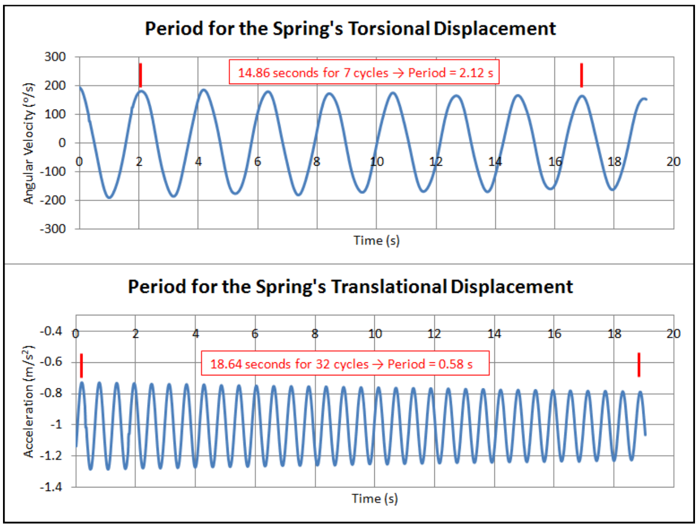

Figure 4

Calculation of the Translational Spring Constant k

A balance is used to measure the mass of the bob, which consists of the wood block and Voyager. With the mass equal to 0.04834 kg and the period found to be 0.58 s, the leftmost equation in Figure 1 can be solved for the spring constant $k = 4\pi^2 m/T^2 = 5.67\ kg/s^2 = 5.67\ N/m$. Therefore, we find that the Vernier nominal 5 N/m spring is closer to 5.67 N/m. It is always a good idea to do an experiment to determine the actual value rather than use the nominal value, as spring constants can change with continued use over time.

However, the main purpose of this experiment is to investigate the torsional oscillation of the spring. Therefore, the remainder of our analysis deals with that.

Calculation of the Torsional Spring Constant κ

In order to compute the torsional spring constant κ, the rightmost equation in Figure 1 indicates that we need to determine the moment of inertia of the bob spinning on its axis, the axis of the vertical spring. Moments of inertia for a variety of common figures can be found by doing a Google search of "table of moments of inertia". Figure 5 shows how to compute the moment of inertia of our bob consisting of the wood block and Voyager. Note that the moment of inertia is independent of the height of the bob. It depends only on the width and depth, which both equal 0.0385 m, the dimensions of Voyager. The mass of the wood block plus Voyager is 0.04834 kg. Using the equation in Figure 5, we calculate the moment of inertia of the bob to be 1.194×10^{-5} kg-m^2.

Our wood block had the same width and depth as Voyager, making our calculation a bit easier. If the dimensions of the wood block are different than Voyager, then the moment of inertia of the bob would simply be the sum of the moment of inertia of the wood block plus the moment of inertia of Voyager.

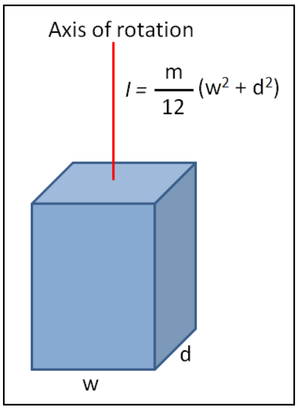

Figure 5

With the moment of inertia *I* of the bob as well as its period *T* now known, it is a straight forward calculation to determine the value of the torsional spring constant κ. We simply solve the rightmost equation in Figure 1 for κ, obtaining $\kappa = 4\pi^2 I/T^2 = 1.05\text{x}10^{-4}$ N-m/rad.

Determining the Moment of Inertia of an Object Experimentally

Once the torsional spring constant is known, it is possible to experimentally determine the moment of inertia of other objects including those with very irregular shapes for which mathematical formulas for the moment of inertia do not exist. Alternatively, you could have students use a regular shaped object as the bob, compute its moment of inertia and compare it to moments of inertia in tables of moments of inertia. The experimental steps would be as follows:

1. Pick an object such as a disk, rod, sphere, *etc.* and decide on what the axis of rotation will be.
2. Use a balance to determine the mass of voyager and the mass of the object.
3. Using the formula in Figure 5, compute the moment of inertia of Voyager $I_{Voyager}$ about the selected axis.
4. Mount Voyager to the object so that its x, y, or z axis is centered on the axis of rotation.
5. Suspend the object plus Voyager from the spring and determine its period *T* from data on angular velocity.
6. Using the rightmost equation in Figure 1 and solving the equation for *I*, compute the moment of inertia $I_{Combined}$ of the combined object and voyager.
7. To get the moment of inertia of the object alone, subtract the moment of inertia of voyager from the combined moments of inertia: $I_{object} = I_{Combined} - I_{Voyager}$.
8. Compare your experimental value with the value from formulas in a table of moments of inertia.
9. How might you account for any differences in the two values?