PocketLab Voyager: The Effect of Area on Terminal Velocity of a Falling Object

By Richard Born Associate Professor Emeritus Northern Illinois University

A common experiment in physics classes involves the use of basket coffee filters to determine how the mass of a falling object relates to its terminal velocity. This is accomplished by varying the number of coffee filters, each having nearly the same mass. Coffee filters were selected since they are light enough to allow reaching a terminal velocity in a relatively short distance.

But how might we investigate the effect of *area* on the terminal velocity of a falling object? Doing this would require us to *control* the mass and the shape. One way to do this is to use a single sheet of 8½ x 11 inch cardstock. Cardstock was found better than thinner paper since, being stiffer, it kept its shape better while falling. While holding the cardstock with its surface horizontal, it is released about one meter above a PocketLab Voyager that is lying on the floor. The range finder is set to 95 points per second. The card stock is then folded in half and released a second time. It is folded in half once again and released a third time. Data from the PocketLab app allows determining the terminal velocity in each of these three cases. We have controlled mass since the entire mass of a single sheet is used for each case, and we have controlled the shape, at least to the extent that we have a *flat surface* in each case. If we take the area after the three folds to be 1 arbitrary unit of area (AUA), then we have cardstock areas of 1, 2, and 4 AUA.

For each of the three cases, it is a good idea to drop the cardstock five times and use the average terminal velocity from the five drops. Data from the PocketLab app can be imported into Excel for further analysis. Figure 1 shows a graph of height (above the floor) vs. time for the case of dropping the cardstock before any folds have been done, i.e., 4 AUA. It is seen that the five drops are easily obtained over a period of less than a minute.





Figure 2 zooms in on the first drop that took place during the first five seconds of data collection. During the first second, the range finder "sees" the ceiling that is about 2.3 meters above Voyager on the floor. Between seconds 3 and 4, the cardstock is held fairly steady at about 1 meter above the floor. During the 4th second, the cardstock is released and quickly reaches a constant terminal speed of 1.09 m/s, obtained from the slope of the region where the cardstock is falling. The average of the five drops turns out to be 1.065 m/s for the cardstock with 4 AUA. The average speed for the 2 AUA 1.439 m/s, and the average speed for the 1 AUA is 1.996 m/sec. Our first qualitative observation is that larger areas have smaller terminal velocities. This is in agreement with what we know, for example, about the behavior of parachutes in contrast to a skydiver in freefall before opening the parachute.





Figure 3 shows a graph of terminal velocity vs. area. Using the "Add Trendline" feature of Excel and selecting "power" as the trend/regression type, we find that the exponent for the independent variable is -0.453. This is very close to -0.5, suggesting the possibility that terminal velocity may be proportional to the square root of the area. The R^2 value is very close to 1, meaning that the data points are in very close agreement to the suggested power relationship.





Further Data Analysis for Students

Models of air resistance (*aka drag*) that include the effects of surface area and velocity typically take the form $F_{drag} = KAv_t^{\rho}$, where K is a constant, A is the surface area, v_t is the terminal velocity, and p is the power relationship used in the model for terminal velocity. At terminal velocity v_t , the magnitude of the drag force F_{drag} equals the weight, i.e., $mg = KAv_t^{\rho}$. Since m and g are constant in our experiment, then the product Av_t^{ρ} is constant. Students can try varying value of p to see which model seems to fit their data the best. In our example experiment of Figure 3, a value of p = 2 provides the best fit.