Graphing Trigonometric Functions using Unit Circle and PocketLab's Accelerometer

Exploration

A unit circle is a circle with a radius of 1, graphed on a Cartesian plane, with the center of the circle at the origin. The relationship between the angle of different radii on the circle and the values of different points on the circle can help in the understanding trigonometric functions.

Objective

In this exploration, students will:

1. Learn how to use the unit circle to graph trigonometric functions.
2. Use the PocketLab's accelerometer when attached to a rotating wheel to visualize how a sine function is graphed from the coordinates of a unit circle.

Materials

- PocketLab
- Rotating wheel
- Tape or velcro to attach PocketLab to wheel

The Unit Circle and the Values of Sine and Cosine Functions

A unit circle is graphed on a Cartesian plane with a radius of 1. The center of the circle is the origin on the graph. When a ray extends from the origin through a point *p* on the Unit Circle the angle formed between the x-axis and the ray is θ. The value of $sin(\theta)$ is equal to the y-coordinate of the point *p* and the value of $cos(\theta)$ equals the x-coordinate of the point p. See figure 1.

The Unit Circle also shows the angles measured in radians and degrees. Start at 0° or 0π and follow the circle in a counterclockwise direction to increase the angle to 90° or π/2. This decreases the cosine, or value of the x-coordinate, because it is closer to the y-axis while increasing the sine, or value of the y-coordinate, because it farther from the x-axis. After one complete revolution the point has moved 360° or 2π. This will be one complete period when graphing sine and cosine functions. See figure 1.

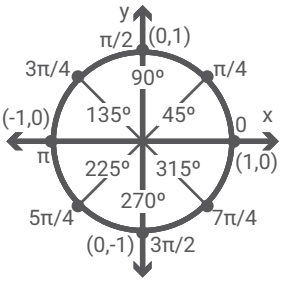


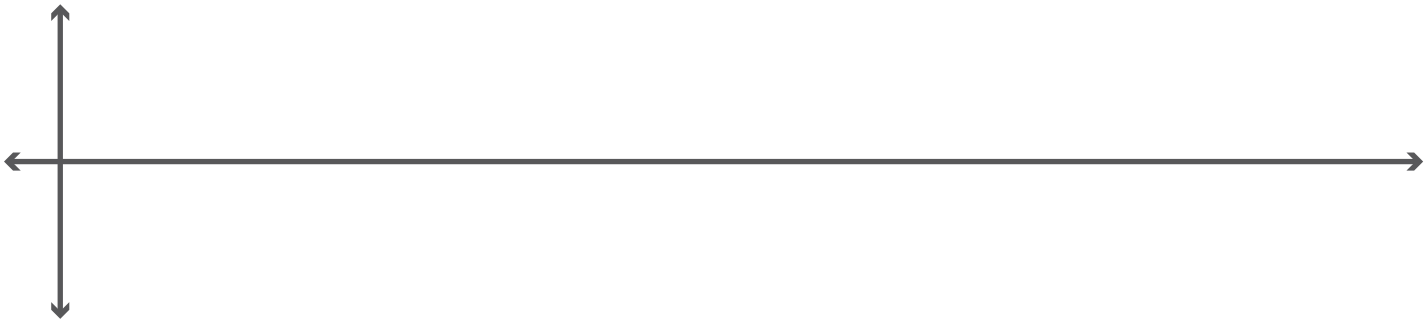
Figure 1

Graphing a sine function *(y=sin(θ))* and a cosine function *(y = cos(θ))*

Set up a table to evaluate the basic sine function. Use the main intervals 0° or 0π, 90° or π/2, 180° or π, 270° or 3π/2, and 360 ° or 2π for the x-values of the function, and the corresponding y-coordinates on the unit circle for the y-values of the function. Graph the function.

$f(x)=sin(\theta)$	
θ	$f(x)$
0° or 0π	0
90° or π/2	1
180° or π/2	0
270° or 3π/2	-1
360° or 2π	0

Set up a table to evaluate the basic cosine function. Use the main intervals 0° or 0π, 90° or π/2, 180° or π, 270° or 3π/2, and 360 ° or 2π for the x-values of the function, and the corresponding x-coordinates on the unit circle for the y-values of the function. Graph the function.



$f(x)=cos(\theta)$	
θ	$f(x)$
0° or 0π	1
90° or π/2	0
180° or π/2	-1
270° or 3π/2	0
360° or 2π	1

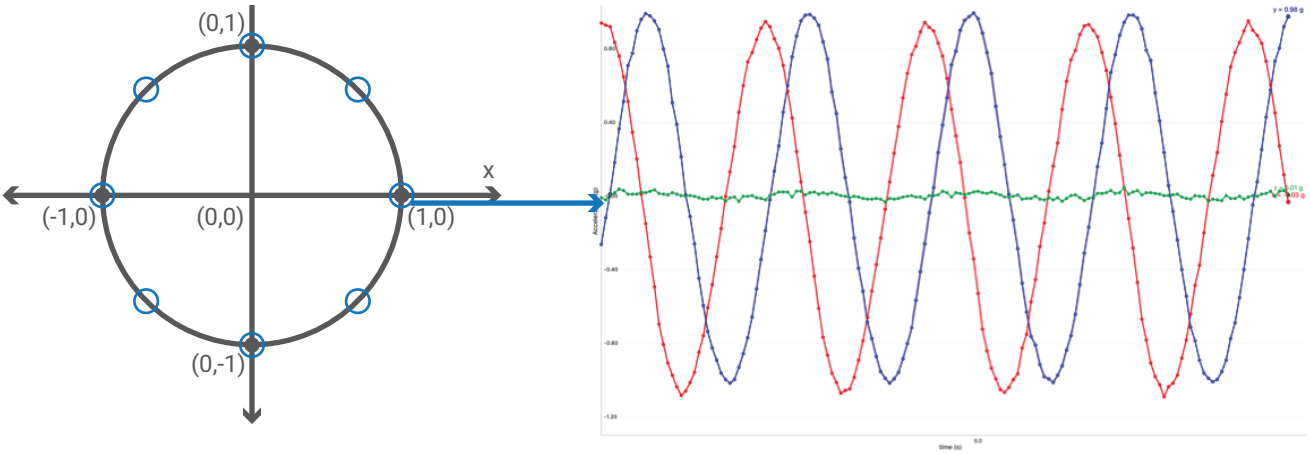
Visualize the relationship between the Unit Circle and sine/cosine waves with PocketLab

1. Place the PocketLab on the rotating wheel oriented so the y-axis of the PocketLab is parallel to the ground and the back of the PocketLab (orange side) is facing you. Draw an arrow from the PocketLab to 0° on the Unit Circle.
2. Rotate the wheel counterclockwise, so the arrow from the PocketLab is moving around the unit circle.
3. In the Acceleration graph, observe the blue (y) and red (x) lines. Observe the patterns they create and the values they display between -1 (g) and +1 (g).

Y-graph (blue)

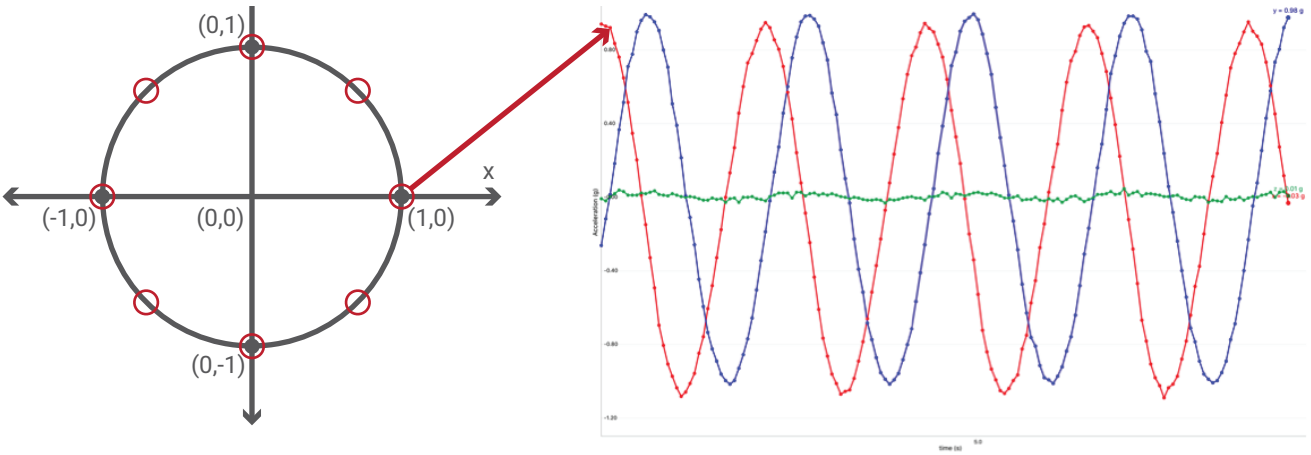
- The blue, y-graph created will be a sine wave.
- The acceleration values (g) on the blue graph correspond to the y-coordinates on the unit circle.

Use the PocketLab's blue, y-graph to help you match the points on the Unit Circle with the points on the sine function, graphed below. Draw a line between the different points on the Unit Circle and the corresponding point on the sine function. The first point is done as an example.



X-graph (red)

- The red, x-graph created will be a cosine wave.
- The acceleration values (g) on the red graph correspond to the x-coordinates on the unit circle.
- Use the PocketLab's red, x-graph to help you match the points on the Unit Circle with the points on the cosine function, graphed below. Draw a line between the different points on the Unit Circle and the corresponding point on the sine function. The first point is done as an example.



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