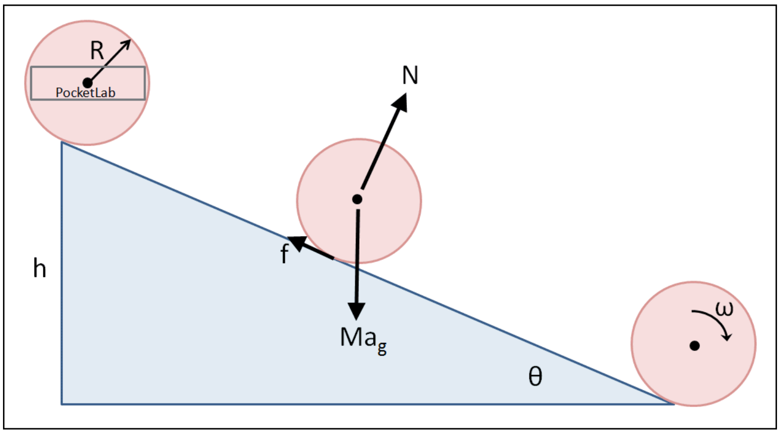
Derivation of Equations



We will use conservation of energy to determine the velocity of the cylinder at the bottom of the incline, though this can also be done from the use of dynamics as well. While rolling down the incline, the cylinder loses Ma_gh of potential energy. Meanwhile, it has gained an amount of kinetic energy equal to

$$\tfrac{1}{2}I_{cm}\omega^2 + \tfrac{1}{2}Mv^2$$

where I_{cm} is the moment of inertia of a solid cylinder about the axis of the cylinder, ω is the angular speed and v is the linear speed of the center of mass at the bottom of the incline. Conservation of energy tells us that

$$Ma_gh = \tfrac{1}{2}I_{cm}\omega^2 + \tfrac{1}{2}Mv^2$$

The moment of inertia of a solid cylinder about the cylinder axis I_{cm} is $\tfrac{1}{2}MR^2$ and $\omega = v/R$. A little bit of algebra tells us that *the speed of the center of mass at the bottom of the incline is*

$$\boxed{v = \sqrt{\frac{4a_gh}{3}}}$$

To determine the translational acceleration of the center of mass of the cylinder at the bottom of the incline we will make use of dynamics. Considering the free body diagram and using Newton's second law of motion, for motion that is normal to the incline

$$N - ma_g \cos\theta = 0.$$

For motion that is parallel to the incline

$$Ma_g \sin\theta - f = Ma,$$

where a is the translational acceleration of the center of mass as it rolls down the incline.

For rotational motion of the center of mass, the torque $\tau = I_{cm}\alpha$, where α is the angular acceleration. Neither the normal force N nor the force of gravity Ma_g can cause rotation since they have zero moment arms. However, the force of friction f does have a moment arm R about the center of mass. Therefore,

$$fR = \tau = I_{cm}\alpha.$$

Since $I_{cm} = \frac{1}{2}MR^2$ and $\alpha = a/R$, and substituting into the equation $Ma_g \sin\theta - f = Ma$, a little algebra reveals that *the translational acceleration of the center of mass of the cylinder is*

$$\boxed{a = \frac{2}{3}\, a_g \sin\theta\,.}$$